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Beta-derivative and sub-equation method applied to the optical solitons in medium with parabolic law nonlinearity and higher order dispersion

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ABSTRACT

By using the sub-equation method, we construct the analytical solutions of the space-time generalized nonlinear Schrödinger equation involving the beta-derivative. This equation describing the propagation of ultra-short optical solitons through parabolic law medium. Nonlinear perturbations of higher-order and self-steepening terms are taken into account. As a result, some new exact solutions are constructed under constraint conditions.

1. Introduction

Recently, many analytical and numerical methods have been proposed to obtain exact solutions of nonlinear partial differential equations (NLPDEs) [1–14]. In the particular case of the sub-equation method [15], we have

$$[G'(\eta)]^2 = q_2G^2(\eta) + q_3G^3(\eta) + q_4G^4(\eta). \tag{1}$$

Now we introduce the following transformation

$$G(\eta) = \frac{1}{g(\eta)}, \tag{2}$$

substituting Eq. (2) into Eq. (1) leads to

$$[g'(\eta)]^2 = q_4 + q_3g(\eta) + q_2g^2(\eta). \tag{3}$$

Derivating Eq. (3) once with respect to η, we get

$$g''(\eta) = q_3g(\eta) + \frac{q_3}{2}. \tag{4}$$

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For Eq. (4) we have the following general solutions

(i)

$$g_1(\eta) = c_1 \exp(\sqrt{q_2}\eta) + c_2 \exp(-\sqrt{q_2}\eta) - \frac{q_3}{2q_2}, \tag{5}$$

where $q_2 > 0$.

(ii)

$$g_2(\eta) = c_1 \cos(\sqrt{-q_2}\eta) + c_2 \sin(\sqrt{-q_2}\eta) - \frac{q_3}{2q_2}, \tag{6}$$

where $q_2 < 0$.

(iii)

$$g_3(\eta) = \frac{q_3}{4}\eta^2 + c_1\eta + c_2, \tag{7}$$

where $q_2 = 0$.

For all cases, q_3, c_1 and c_2 are arbitrary constants. The general solutions of Eq. (1) are obtained substituting Eqs. (5)–(7) into Eq. (2)

Case A:

$$G_1(\eta) = \frac{1}{c_1 \exp(\sqrt{q_2}\eta) + c_2 \exp(-\sqrt{q_2}\eta) - \frac{q_3}{2q_2}}, \tag{8}$$

where $q_2 > 0$, $4q_4q_2 + 16q_2^2c_1c_2 = q_3^2$.

Case B:

$$G_2(\eta) = \frac{1}{c_1 \cos(\sqrt{-q_2}\eta) + c_2 \sin(\sqrt{-q_2}\eta) - \frac{q_3}{2q_2}}, \tag{9}$$

where $q_2 < 0$, $4q_4q_2 + 4q_2^2(c_1^2 + c_2^2) = q_3^2$.

Case C:

$$G_3(\eta) = \frac{1}{\frac{q_3}{4}\eta^2 + c_1\eta + c_2}, \tag{10}$$

where $c_1^2 = q_3c_2 + q_4$, when $q_2 = 0$ in Eq. (1).

The nonlinear Schrödinger equations (NLSE) describing the propagation of optical pulse in nonlinear media [16–25]. The exact solutions of these equations are important in optics, optical communication areas, propagation of optical pulses in optical fibers, ultra-short optical solitons propagate in nonlinear medium, electromagnetism, plasma and fluids. Furthermore, the exact solutions can be used to explain various phenomena in physics or other areas.

The concept of memory effect has long been a problem within the modeling community. Naturally, the classical models are not appropriate to incorporate this memory [26–28]. Many researchers suggest that the memory effect could fully be described via fractional derivatives [29–32]. In [33] Khalil presented a new definition of derivative called "conformable derivative", this derivative satisfied some conventional properties, for instance, the chain rule. Atangana in [34] investigated some properties of this derivative, the authors proved related theorems and introduced new definitions. Interesting works related with this operator are given by [35–39]. Recently Abdon Atangana in [40] proposed the "beta-derivative". The version proposed satisfies several properties that were as limitation for the fractional derivatives and has been used to model some physical problems. These derivatives may not be seen as fractional derivative but can be considered to be a natural extension of the classical derivative [33].

The beta-derivative is defined as [40]

$${}_0^AD_x^\alpha\{f(x)\} = \lim_{\epsilon\to 0} \frac{f\left(x + \epsilon\left(x + \frac{1}{\Gamma(\alpha)}\right)^{1-\alpha}\right) - f(x)}{\epsilon}. \tag{11}$$

Some properties for the proposed beta-derivative are [40]

(I) Assuming that, a and b are real numbers, $g \neq 0$ and f are two functions β-differentiable and $\beta \in (0;1]$, we have

$${}_0^AD_x^\alpha\{af(x) + bg(x)\} = a\,{}_0^AD_x^\alpha f(x) + b\,{}_0^AD_x^\alpha g(x). \tag{12}$$

(II)

$${}_0^AD_x^\alpha\{c\} = 0, \tag{13}$$

for c any given constant.

(III)

$${}_0^AD_x^\alpha\{f(x)\cdot g(x)\} = g(x)\,{}_0^AD_x^\alpha\{f(x)\} + f(x)\,{}_0^AD_x^\alpha\{g(x)\}. \tag{14}$$