Optik 155 (2018) 251–256

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Short note

Comments on "Shilnikov chaos and Hopf bifurcation in three-dimensional differential system"

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ARTICLE INFO

Article history:
Received 2 September 2017
Accepted 2 November 2017

Keywords:
3D differential system
Shilnikov criterion
Homoclinic orbit
Smale horseshoe
Undetermined coefficient method

ABSTRACT

In the commented paper, the authors consider a three-dimensional system and analyze the presence of Shilnikov chaos as well as a Hopf bifurcation. On the one hand, they state that the existence of a chaotic attractor is verified via the homoclinic Shilnikov theorem. The homoclinic orbit of this system is determined by using the undetermined coefficient method, introduced by Zhou et al. in [Chen's attractor exists, Int. J. Bifurcation Chaos 14 (2004) 3167–3178], a paper that presents very serious shortcomings. However, it has been cited dozens of times and its erroneous method has been copied in lots of papers, including the commented paper where an even expression for the first component of the homoclinic connection is used. It is evident that this even expression cannot represent the first component of a Shilnikov homoclinic connection, an orbit which is necessarily non-symmetric. Consequently, the results stated in Section 3, the core of the paper, are worthless. On the other hand, the study of the Hopf bifurcation presented in Section 4 is also wrong because the first Lyapunov coefficient provided by the authors is incorrect.

1. Introduction

In the commented paper the authors consider the system

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = -px - qy - z + y^2 - xy, \end{cases} \tag{1}$$

where p and q are positive real parameters. It has an equilibrium point at the origin, $O=(0, 0, 0)$, for all parameter values. The authors analyze both the existence of Shilnikov chaos and a Hopf bifurcation.

On the one hand, they claim to have rigorously proved the existence of Shilnikov chaos in this system. Thus, we find the following statements along the paper: "*The existence of chaotic attractor in the system is verified via the homoclinic Silnikov*

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https://doi.org/10.1016/j.ijleo.2017.11.018
0030-4026/

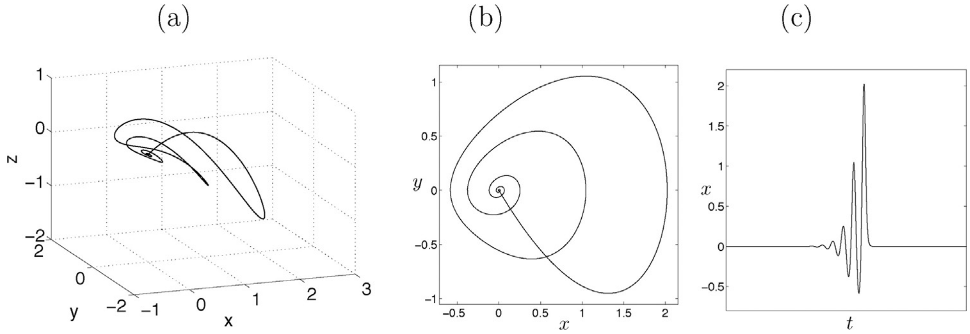

Fig. 1. Shilnikov homoclinic orbit to the origin O of the system (1) for $p \approx 0.699556$, $q = 0.15$: (a) 3D view in the phase space; (b) projection onto the x-y plane; (c) first component $x(t)$ of the homoclinic orbit.

theorem. By using the undetermined coefficient method, the homoclinic orbit of the system is determined" (see); "*The Shilnikov chaos and Hopf bifurcation of the system are presented. The homoclinic orbit is determined by using the undetermined coefficient method. Based on the Shilnikov theorem, the horseshoe chaos of the system is verified*" (see); "*In this section, the Shilnikov theorem is introduced for giving a rigorous mathematical proof of the existence of chaotic attractor in the system (1). By using the undetermined coefficient method, the homoclinic orbit of system (1) is determined*" (see); "*The homoclinic orbit of the system is found by applying the undetermined coefficient method. Based on the Shilnikov theorem, a mathematical proof of the existence of the horseshoe chaos in the system is presented*" (see);

The Shilnikov criterion the authors use (see) guarantees the existence of horseshoe chaos when a homoclinic orbit joins a saddle-focus equilibrium, whose eigenvalues satisfy the Shilnikov inequality.

Thus, to prove the existence of the Shilnikov homoclinic orbit, the authors use the undetermined coefficient method. This method was first introduced by Zhou et al. [2], to analytically demonstrate that the Chen system has Shilnikov homoclinic and heteroclinic orbits. Unfortunately, although this method begins with an erroneous assumption and reaches an absurd final result (see, for instance, [3–13]), it is being used in the literature for more than one decade (see, for example, and, also, many of the references cited in [8,9]). Moreover, surprisingly, the papers using this flawed method continue to be cited nowadays as if they were correct.

Lamentably, as we are going to make clear along this comment, the starting point of the procedure used in [1] is erroneous since the authors use an even function to represent a component of the Shilnikov homoclinic orbit. Moreover, the result achieved with the undetermined coefficient method (see) is unreasonable: the authors state that the system considered has a Shilnikov homoclinic orbit provided that the equilibrium is saddle-focus satisfying the Shilnikov inequality. Consequently, they have found Shilnikov homoclinic orbits of codimension-zero that are ubiquitous in an open set of the parameter space of the system.

On the other hand, the Hopf bifurcation of the equilibrium at the origin of system (1) is also studied (see). Unfortunately, this analysis is invalid too because they affirm that there is a degeneration where the Hopf bifurcation changes from subcritical to supercritical whereas this bifurcation is always supercritical.

This comment is organized as follows. In Section 2 the flaws of the modus operandi followed in [1] in the study of the homoclinic orbit of system (1) are pointed out. Section 3 is devoted to bring out the mistakes of [1] in the analysis of the Hopf bifurcation of the origin. Some concluding remarks are provided in Section 4.

2. Invalid homoclinic analysis

In, the authors study homoclinic orbits to the origin O, in a parameter region where it is a saddle-focus, with negative real eigenvalue and positive real part of the complex pair. We have numerically found using AUTO [14] a Shilnikov homoclinic orbit in the system (1), that exists when $p \approx 0.699556$, $q = 0.15$ (see Fig. 1). Thus, they want to determine a Shilnikov homoclinic orbit that must have the shape sketched in Fig. 2(a) and (b).

They apply the undetermined coefficient method to find an expression of the first component of the homoclinic orbit. First, they convert system (1) into a third-order equation for $x(t)$. Thus, in order to find a homoclinic orbit, it is enough to determine a solution $x(t)$ that tends to O when $t \to \pm\infty$.

To do that, they consider that the first component of the homoclinic orbit connecting O takes the following form

$$x(t) = \psi(t) = \sum_{k=1}^{\infty} a_k e^{k\alpha t}, \qquad t > 0.$$

Then, after some reasonings and tedious computations they obtain that α is determined by the parameters p, q (in fact, α is the negative eigenvalue associated to O) and the coefficients a_k $(k \geq 2)$ are completely fixed by p, q, α and a_1. Moreover,