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Rapid Determination of the Fatigue Limit by the Simulation of Self-Heating Test by the Collaborative Model Based On the Fractional Derivative Approach

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Abstract

This paper proposes a method for on the fast fatigue limit estimation for composite materials using a simulation of a self-heating test. The experimental method based on the material self-heating was successfully applied for composite materials and allows to determine a material's endurance limit in a few hours. In order to increase the potential of this experience, the thermomechanical modelling is proposed with the precise description of the intrinsic energy dissipation. The rise of material's temperature is linked with development of plastic strains, material damage, viscoelastic proprieties of polymer matrix and intro-crack friction. Therefore, the collaborative behaviour model including the hysteresis loops is used to represent the visco-elastoplastic damage composite behaviour. The collaborative model consists of two sub-models. The first one describes an envelope of the loading curves and insures the computation of elastic and the in-elastic strains as well as the in-ply damage. The second part deals with modelling of hysteresis loops during unloading path using a fractional derivative approach. Just a few parameters are required to represent the hysteresis loops. Using a proposed modelling, the dissipation due to the in-ply damage propagation, the material hardening and the viscoelastic effects can be precisely calculated. The fatigue limit is determined by using thermodynamic simulation the quasi-static cyclic test with the dissipation measurements as an equivalent to self-heating test. The method is validated for composite materials with thermosetting and thermoplastic matrixes.

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Keywords: Composite ; Fatigue ; Self-heating ; Dissipation ; Behaviour modelling ; Fractional derivative ; Hysteresis loop

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1. Introduction

Nowadays, carbon fiber reinforced plastics (CFRP) are widely used in the different industrial fields. Their lightweight, high strength and long durability make them superior to classical metallic materials in the complex structures design. In order to increase the structure's safety and their economic potential, the validation phase has to be made including the mechanical test and numerical simulations of the material behaviour. In the present work, the fatigue loading is concerned as the essential point in the certification of the industrial structures.

The experimental measurement of fatigue limit for composite materials required a significant number of tests in the different orthotropic directions. The classical method consists in the applying of the cyclic loading for the given laminate during a substantial period of time (usually several weeks) in order to obtain Wöhler curve. The alternative method allows to determine the fatigue limit in several hours using the material self-heating effect. This method has been initially proposed for isotropic materials such as metal alloys [1], elastomers [2] and more recently for the short fiber composite materials [3]. In the case of carbon fiber composites, the self-heating method consists of applying a sequence of constant amplitude cyclic loading blocks. The stabilized temperature of the composite specimen is measured during each block. When the value of the stabilized temperature increases significantly, it is considered that the fatigue limit is attainted. Tomographic studies and the traditional fatigue tests allow to justify the developed method. The experimental results of the self-heating method are in good agreement with the conventional fatigue tests (Wöhler curve) for thermosetting and thermoplastic composite materials [4], [5].

The potential of the experimental method can be increased by using the numerical simulation of self-heating test proposed in this paper. In order to calculate the variation of material's temperature, a full thermo-mechanical material analysis is required. The collaborative model [6] is a suitable tool to calculate the intrinsic dissipation including the hysteresis loops which determine the quantity of dissipated energy dominating in the material self-heating. The proposed model is composed of the elastoplastic damage behaviour law [7] with possible strain-rate sensitivity [8], [9] and including a newly proposed fractional derivative approach.

A fractional derivative theory is a promising technique to describe the history-dependent phenomena as hysteresis loops. The viscoelastic equation is one of the first applications of fractional calculus. The fractional derivatives provide the formulation of the behaviour law in the integral form thus ensure the contribution of the past loading history. Considering that the material is pristine in the initial state $\varepsilon(0) = 0$, the hereditary stress-strain relation for polymer materials can be represented by the integral law (1) [10], [11]:

$$\sigma(t) = \frac{G}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{\varepsilon(\tau)}{(t-\tau)^{\alpha}} d\tau = G D_0^{\alpha} \varepsilon(t) \tag{1}$$

where G is the material parameter, Γ is the gamma function defined by equation (3) and D_0^{α} signifies a fractional derivative of order α (4).

In the rheological sense, the fractional derivative operator D_0^{α} can be represented by the spring-pot element [12]. This element is an asymptotic representation of the assembly of the elastic (spring) and viscoelastic (dash-pot) elements connected in the series and parallel (Figure 1). The spring-pot element is able to capture different types of the mechanical behaviour by the variation of order α. If the fractional derivative order tends to zero: $\alpha \to 0$, the behaviour of the element tends to the elastic spring and if the fractional derivative order tends to one: $\alpha \to 1$, the viscoelasticity increases.

From a mathematical point of view, the fractional operators have several definitions. In this work the classical Riemann-Liouville definition is used [13]. The fractional Riemann-Liouville integral of order α is defined as following:

$$(I_a^{\alpha} f)_{RL}(t) \overset{\text{def}}{=} \frac{1}{\Gamma(\alpha)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \quad \alpha > 0 \tag{2}$$