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ScienceDirect

Procedia Engineering 203 (2017) 78–90

Procedia Engineering

www.elsevier.com/locate/procedia

26th International Meshing Roundtable, IMR26, 18-21 September 2017, Barcelona, Spain

3D Metric-aligned and orthogonal solution adaptive mesh generation

D. Marcum[a,*], F. Alauzet[b]

[a] *Center for Advanced Vehicular Systems and Dept. of Mechanical Engineering, Mississippi State University, USA, marcum@cavs.msstate.edu*
[b] *Gamma3 Team, INRIA Saclay, France, Frederic.Alauzet@inria.fr*

Abstract

A new 3D metric-orthogonal method for metric-based anisotropic tetrahedral mesh generation is presented. The approach uses an advancing-point type point placement that aligns the elements with the underlying metric field and creates quasi-structured local topology. It is based on well-proven methodology and adds novel point-placement strategies that produce solution-adapted anisotropic meshes that are optimal in size and alignment with the solution gradients upon which the metric field is based. The overall methodology is extended to 3D from a previously developed 2D process. Results for several analytical and actual CFD simulation examples are presented that demonstrate that aligned high-quality anisotropic tetrahedral meshes which are optimal for the solution process can be reliably generated.

Peer-review under responsibility of the scientific committee of the 26th International Meshing Roundtable.

Keywords: Unstructured Mesh Generation; Metric-based Mesh Adaptation; Metric-Aligned; Metric-Orthogonal; Advancing Front; Local-Reconnection

1. Introduction

The benefits and fundamental concepts of metric-based mesh adaptation for dealing with anisotropic physical phenomena are well established [9]. Several successful examples [1,2,4] with real-life problems have proven its ability to efficiently improve the ratio between solution accuracy and the number of degrees of freedom. This success has been based on the following key points:

- Efficient adaptive anisotropic mesh generator that can handle extreme anisotropy
- Accurate metric-based anisotropic error estimates: feature- or adjoint-based
- Appropriate operator on metrics: interpolation, intersection and gradation
- Accurate solution transfer for transient problems

There are several solver and meshing tools that utilize a solution adaptive metric-based concept. Some examples in 3D include EPIC[19], Feflo.a[10], FUN3D[6], and MeshAdapt[7]. It is worth mentioning that many of these codes include adaptive remeshers based on local mesh modifications and others are mesh generation codes using

*Corresponding author.
E-mail address: marcum@cavs.msstate.edu

1877-7058
Peer-review under responsibility of the scientific committee of the 26th International Meshing Roundtable.
10.1016/j.proeng.2017.09.790

Delaunay kernels or related approaches. Existing approaches assume that mesh quality and edge length sufficiency are fully satisfied if true in metric space. However, element shape and alignment with the metric field also impacts accuracy and efficiency of the solution process. In addition, during mesh generation without alignment, a local loss of anisotropy can occur when an element is generated that is not aligned (as an ideal equilateral element in metric space can vary in shape from isotropic to highly anisotropic as it is rotated). There is a need to investigate approaches that can provide improvement in this aspect and the present work seeks to do that in three dimensions.

The approach used in this work is based on the existing AFLR advancing-front method with local-reconnection (edge/face-swapping) for mesh connectivity optimization [14,15,17], which is known to generate very-high quality unstructured meshes as compared to Delaunay type approaches. A new extension of this methodology to metric-based anisotropic mesh generation is used in our approach, wherein the overall process is modified to satisfy a metric space definition. All geometric operations are performed in metric space with a consistent operational framework as defined in [9].

In addition a novel point placement algorithm has been developed to align the mesh elements with the solution based metric field. When new points are created using the advancing-front point placement they are by default aligned in physical space with the face that locally represents the front. In metric space the ideal point placement is chosen to form tetrahedral elements that are ideal in metric space and aligned in physical space. In our approach the alignment is performed with the local metric field, rather than solely with the frontal face. We will refer to this approach as metric-aligned anisotropic mesh adaptation. Further improvements can be obtained if we consider local topology. Using right-angle type advancing-point point placement [14], quasi-structured element topology is produced locally that can be fully aligned with the solution gradients by the metric field. With right-angle type placement three-points are created from the face that locally represents the front. Alignment with the metric produces elements that are naturally ideal with respect to the metric field and also have quasi-structured topology. We will refer to this approach as metric-orthogonal anisotropic mesh adaptation.

This methodology has been successfully implemented in the 2D version of AFLR [12]. The results obtained are of surprisingly good quality with respect to element shape as well as alignment with the metric field. A first extension has been done in 3D in Feflo.a within a different context and using a modified approach with a cavity-based local remeshing algorithm [11].

The extension of the metric-aligned and metric-orthogonal methods in three-dimensions for the advancing front method is natural as the point placement strategies are following the same algorithm as in two-dimensions. In three-dimensions, AFLR uses local-reconnection for connectivity optimization with a combined Delaunay and max angle minimization type criterion, to minimize slivers (nearly degenerate elements) that are encountered in pure Delaunay-type methods. This optimization is especially important in the three-dimensional metric-orthogonal approach due to quasi-structured element formations. Moreover, based on the experience of the authors in boundary layer meshing [3,13,16] it is possible to extract mixed-element type meshes from this purely unstructured strategy when alignment is considered. This hopefully will eventually lead to a fully automatic mesh generation method capable of generating quasi-structured adapted meshes.

2. Metric-based anisotropic mesh generation

We utilize the concept of metric-based mesh generation, initially introduced in [5], to generate a solution adaptive anisotropic mesh. The details, in the context of the present work, are presented in [3,11,12]. All geometrical operations are performed in either Euclidean metric-space (volume, angles, etc.) or a Riemannian metric-space (distance).

2.1. Euclidean metric space

For the sake of clarity, we recall the differential geometry notions that are used in the sequel. We use the following notations: bold face symbols, as $\mathbf{a}, \mathbf{b}, \mathbf{u}, \mathbf{v}, \mathbf{x}, \ldots$, denote vectors or points of $\mathbb{R}^3$. The natural dot and cross product between two vectors $\mathbf{u}$ and $\mathbf{v}$ of $\mathbb{R}^3$ are denoted by: $\mathbf{u} \cdot \mathbf{v} = \langle \mathbf{u},\ \mathbf{v} \rangle$ and $\mathbf{u} \times \mathbf{v}$.

A **Euclidean metric space** $(\mathbb{R}^3, \mathcal{M})$ is a vector space of finite dimension where the dot product is defined by means of a symmetric definite positive tensor $\mathcal{M}$:

$$\mathbf{u} \cdot_{\mathcal{M}} \mathbf{v} = \langle \mathbf{u}, \mathbf{v} \rangle_{\mathcal{M}} = \langle \mathbf{u}, \mathcal{M}\mathbf{v} \rangle = {}^t\mathbf{u}\, \mathcal{M}\, \mathbf{v}\,, \quad \text{for } (\mathbf{u}, \mathbf{v}) \in \mathbb{R}^3 \times \mathbb{R}^3\,. \tag{1}$$