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An augmented Lagrangian formulation to impose boundary conditions for distortion based mesh moving and curving

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Abstract

We formulate a mesh morphing technique as mesh distortion minimization problem constrained to weakly satisfy the imposed displacement of the boundary nodes. The method is devised to penalize the appearance of inverted elements during the optimization process. Accordingly, we have not equipped the method with untangling capabilities. To solve the constrained minimization problem, we apply the augmented Lagrangian technique to incorporate the boundary condition in the objective function using the Lagrangian multipliers and a penalty parameter. We have applied the proposed formulation to mesh moving and mesh curving problems. The results show that the method has the ability to deal with large displacements for 2D and 3D meshes with non-uniform sizing, and mesh curving of highly stretched 2D high-order meshes.

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Keywords: Mesh morphing; mesh moving; mesh curving; smoothing; boundary displacements; augmented Lagrangian

1. Introduction

In several applications such as large deformations and high-order mesh curving techniques, it is needed to deform an initial mesh in order to adapt it to a new configuration determined by a boundary displacement. Usually, it is necessary to recover a valid mesh in which the position of the boundary nodes is the prescribed one. There are different techniques to morph [1] a valid mesh, based on solid mechanics analogies [2–4], optimization-based methods [5–12], and solution of PDE's [13–15].

One critical step is how to impose the boundary displacement condition for the mesh morphing problem. When the mesh is subject to large displacements, the techniques to perform the mesh morphing process may not converge if inverted elements are originated during the procedure. In particular, the mesh morphing technique may require the capability of untangling to recover from invalid configurations and obtain a final valid mesh. For instance, in mesh distortion minimization methods [5,6,9,10], it is necessary to use a regularized distortion measure [6,16] to remove the singularities of the objective function. In [11,12], it is introduced a moving log-barrier to repair the inverted elements.

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In [3,4] the authors propose to incrementally move the mesh boundary. Other methods solve a non-linear PDE [13,14] in which the initial condition of the boundary nodes may impact the performance of the non-linear solver. Note that these different treatments arise because of the imposition of the boundary condition.

In this work, we propose to incorporate the boundary condition into a minimization problem. To this end, we pose a constrained minimization problem in which we optimize the mesh distortion, constrained to the boundary condition. To solve this non-linear minimization problem, we apply the augmented Lagrangian method [17], in which the boundary condition is incorporated into the target function by means of the Lagrangian multipliers and a penalty parameter. Then, we solve a series of minimization problems with increasing penalty parameter and better approximation of the Lagrangian multipliers. The algorithm finishes when the boundary constraint is satisfied within a prescribed tolerance.

The proposed method has several advantages. The mesh boundary is not fixed during the optimization process since it is driven by the penalty parameter and the Lagrange multipliers. Thus, similarly to the incremental moving methods, the boundary condition is only satisfied at the end of the optimization process. However, it is worth to point out that in our method we do not impose the trajectory of the boundary nodes during the deformation process, since it is automatically computed by the augmented Lagrangian method. Moreover, since the mesh is valid during the whole minimization, it is not necessary to feature untangling capabilities. We propose to use a global non-linear solver, in which the nodes of the mesh are moved at the same time. To this end, we use a backtracking line-search method [17] in which the advancing direction is computing using Newton's method and the step length is selected according to the Wolfe condition. We point out that to perform Newton's method we use the analytical derivatives of the objective function.

The rest of the paper is structured as follows. In Section 2, we review the existing literature related to the presented work. In Section 3, we formulate the constrained minimization problem and detail the implemented augmented Lagrangian method. In Section 4 ,we present several examples to show the features of the proposed method. Finally, in Section 5, we detail the conclusions and the future work.

2. Related work

In some applications, mesh untangling and smoothing can be required to repair invalid elements and improve the overall mesh quality when applying a mesh morphing technique. In this cases, the boundary nodes are displaced to a new position, and it is necessary to recover a new mesh composed of valid elements. There are different formulations to define a mesh morphing technique. For instance, in the solid mechanics approach, the mesh is moved using linear or non-linear elasticity analogies [2–4]. Other approaches consist on minimizing an objective function that measures a quantity of interest of the mesh, such as the mesh distortion [5,6,18], the minimum Jacobian of the iso-parametric mapping [11], or by formulating a variational minimization problem [14]. Finally, it is possible to define the mesh morphing process in terms of a PDE [13]. In all these approaches, it is necessary to fix the position of the boundary nodes in order to satisfy the prescribed mesh morphing displacement. Usually, it is introduced as a boundary condition for the mesh morphing problem. In the most common approach, the position of the boundary nodes is fixed, and then, the location of inner nodes is computed accordingly. However, when the boundary nodes are displaced in the initial step, invalid elements may appear, and they could hinder the convergence of the mesh morphing method, specially when complex geometries or non-uniform element sizes are present.

In reference [19], the authors propose to solve a constrained minimization problem in which the constraint is the final position of the boundary nodes. To this end, they apply a penalty method to solve a series of optimization problems to enforce the constraint. Other approaches first project the boundary nodes to the CAD model, and then pose an unconstrained minimization problem in which the boundary nodes can slide along the geometric entity they belong to [10,11,20]. In this case, the imposition of the boundary condition is more flexible than prescribing a fixed nodal value. The only requisite is that the boundary nodes are located on the required geometric entity. In [12], the authors combine a mesh curving technique with a geometric accuracy measure into a single functional. The boundary nodes are free to slide along the geometric entities taking into account the geometric accuracy.

In reference [21], the authors introduce a geometric accuracy measure, and it is later combined with a mesh curving technique in [22]. In this approach, all the nodes of the mesh are free to move in the space, while the boundary nodes take into account the geometric error. In the proposed optimization process, the geometric accuracy term is treated as a