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Hyperelastic springback technique for construction of prismatic mesh layers

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Abstract

We consider an algorithm for construction of thick prismatic mesh layers which works as follows. A triangular surface mesh is specified as input. Then, thin initial layer of highly compressed hyperelastic material glued to the surface is constructed using robust algorithm for computation of discrete normals. This pre-stressed material expands, possibly with self-penetration and extrusion to exterior of computational domain. Special preconditioned relaxation procedure is proposed based on the solution of stationary springback problem. It is shown that preconditioner can handle very stiff problems related to construction of very thick one-cell-wide layers for rather fine surface meshes. Once an offset prismatic mesh is constructed self-intersections are then eliminated using iterative prism cutting procedure. Next, variational advancing front procedure is applied for refinement and precise orthogonalization of prismatic layer near boundaries. It is guaranteed that the resulting mesh is free from inverted prisms.

Keywords: prismatic mesh layer; variational method; hyperelastic deformation; mesh untangling and optmization; preconditioned gradient search

Introduction

High quality simulation of viscous flows imposes rather strict requirements on computational meshes near solid boundaries. It it very important to construct meshes which provide orthogonality near boundary and precise control over mesh element size in the direction orthogonal to boundary irrespectively of the size and shape of surface mesh elements. Variational methods make this precise control possible [1]. Prismatic mesh layers consisting of triangular prisms, hexahedra or general polygonal prisms are flexible enough to be incorporated into automatic mesh generators while providing high quality mesh near boundaries. We consider semi-structured layers with the same mesh connectivity on each sublayer. In literature, sometimes more general case is considered where topology changes are admitted for mesh quality improvement [2]. However, we do not consider this case. Prismatic mesh layer is considered to be "thick" when its transverse size is comparable to the characteristic size of the geometric model. One can also call prismatic layer thick when its height is considerably larger compared to mesh element size on the surface.

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1. Variational principle for construction of prismatic layers

Let ξ_1, ξ_2, ξ_3 denote the Lagrangian coordinates associated with elastic material, and x_1, x_2, x_3 denote the Eulerian coordinates of a material point. Spatial mapping $x(\xi)$: $\mathbb{R}^3 \to \mathbb{R}^3$ defines a stationary elastic deformation. The Jacobian matrix of the mapping $x(\xi)$ is denoted by C, where $c_{ij} = \partial x_i / \partial \xi_j$.

We look for the elastic deformation $x(\xi)$ that minimizes the following weighted stored energy functional [4]

$$F(x) = \int\limits_{\Omega_\xi} w(\xi) W(C)\, d\xi, \tag{1}$$

where $W(C)$ is polyconvex elastic potential (internal energy) which is a weighted sum of shape destortion measure and volume distortion measure [8]:

$$W(C) = (1-\theta) \frac{\left(\frac{1}{3}\operatorname{tr}(C^T C)\right)^{3/2}}{\det C} + \frac{1}{2}\theta\left(\frac{1}{\det C} + \det C\right) \tag{2}$$

In most cases we set $\theta = 4/5$.

Since distortion measure (2) is minimized on the average, locally it can be quite large. In theory it can be infinite on the set of zero measure. In practice it means that with mesh refinement quality of mesh cell can locally deteriorate.

In practice, one can control the spatial distribution of distortion measure without actual contraction of the set of feasible mappings. Experience suggests that large values of distortion appear near boundaries and surfaces of material discontinuity Hence it is possible to introduce a weight function $w(\cdot)$ in the Lagrangian or Eulerian coordinates which takes large values in critical regions and is close to unity elsewhere.

In the process of minimization, elements with a larger weight tend to have a smaller value of distortion function $W(C)$. Hence, their shapes and sizes are very close to the target ones. This simple approach proved to be very efficient for mesh orthogonalization near the boundary [8]. A proper choice of the weight allows us to satisfy the no-slip boundary conditions and to approximate boundary orthogonality conditions and prescribed mesh element size in the normal direction very accurately.

Theoretical arguments suggest that in order to eliminate the local singularities of the distortion function the weight distribution should be singular. However, this singularity is only reached in the limit of mesh refinement and for any given finite mesh weight distribution is bounded. One cannot prove that resulting deformation is quasi-isometric as in [8], [4] but numerical evidences suggest independence of the global mesh distortion bounds from the mesh size.

Suppose that domain Ω_ξ can be partitioned into convex polyhedra U_k. Then stored energy functional (1) can be approximated by the following semi-discrete functional:

$$F(x_h(\xi)) = \sum_k \int\limits_{U_k} w(\xi) W(\nabla x_h(\xi))\, d\xi \tag{3}$$

where $x_h(\xi)$ is continuous piecewise-smooth deformation.

In order to approximate integral over a convex cell U_k one should use certain quadrature rules. As a result semi-discrete functional (3) is replaced by the discrete functional:

$$F(x_h(\xi)) \approx \sum_k \operatorname{vol}(U_k) \sum_{q=1}^{N_k} \beta_q w_q W(C_q) = F^h(x_h(\xi))$$

Here N_k is the number of quadrature nodes per cell U_k, C_q denotes the Jacobian matrix in q-th quadrature node of U_k, while β_q are the quadrature weights and w_q are values of weight function in the quadrature nodes.

The following majorization property should hold

$$F(x_h(\xi)) \le F^h(x_h(\xi)) \tag{4}$$

This property can be used to prove that all intermediate deformations $x_h(\xi)$ providing finite values of discrete functional are homeomorphisms [4].