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Estimation of directional spectra from wave buoys for model validation

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Abstract

In this paper, we consider the problem of estimating a directional wave spectrum from 3-dimensional displacement data recorded by a wave buoy. We look at some of the limitations of existing methods to extend the "first five" directional moments directly obtainable from such data. With a view to providing the most detailed possible comparisons with directional spectra obtained from numerical models, we propose the use of a "diagnostic" directional spectrum, defined to be the closest possible spectrum to a given model spectrum that satisfies all measured directional moments. This method allows us to quantify the *minimum* error in a modelled directional spectrum consistent with a buoy record.

The new method is tested on a range of artificial test cases, and applied to data obtained from a wave buoy deployment off the New Zealand coast, in conjunction with outputs from a numerical spectral wave model simulation. It is shown that the method can provide satisfactory results in a wide range of conditions. Unlike existing approaches, the proposed method can accommodate sea states with more than two directional peaks, and can assist in removing spurious spectral energy arising from existing methods for estimating directional spectra from buoy data.

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1. Introduction

Wave conditions in the ocean can be most thoroughly characterised by combining numerical modelling with measurements. *In situ* measurements from wave buoys are most commonly applied to model evaluation through comparison of derived wave statistics (e.g. significant wave height, mean wave direction, peak wave period). But a more thorough comparison of measurements with full directional spectra predicted by modern wave models could

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offer greater diagnostic insight, allowing us, for example, to separately compare each component of a complex sea state.

Directional wave buoys work by making simultaneous measurements of a small number of signals, e.g. displacements in each of the 3 spatial dimensions. Fourier co- and cross-spectra between pairs of these signals can then be used to provide a limited estimate of the directional spectrum. That is, only the "first five" parameters moments of the directional distribution are obtained, which is insufficient for a full comparison with model spectra that typically have tens of directional degrees of freedom.

In the following section of this paper we review some existing approaches to the problem of extending beyond the measured "first five" moments to estimate a more complete directional wave spectrum from data. We then, in Section 3, propose a new method, which defines a "diagnostic" directional spectrum $S^{(d)}(f,\theta)$ for comparison with a given model directional spectrum $S^{(m)}(f,\theta)$, such that $S^{(d)}$ is the closest possible spectrum to $S^{(m)}$ that satisfies all measured directional moments. This method allows us to quantify the *minimum* error in a modelled directional spectrum consistent with a buoy record. The method is then applied firstly to some artificial test cases, then to directional data obtained from a wave buoy, in conjunction with outputs from a spectral wave forecast. A brief discussion of these results is then given in Section 4.

2. Fourier representation of sea state

Wave motions can be measured by a floating buoy, such as a Waverider™, that records accelerations in up to three dimensions, from which displacements can be computed by double integration of the signal.

First we consider a non-directional instrument that records only vertical motions. The vertical displacement Δz of the sea surface at a fixed position and time t can be represented as a sum of sinusoidal signals:

$$\Delta z(t) = \sum_{m=1}^{M} A_m \cos[-2\pi f_m t + \phi_m] \tag{1}$$

Here A_m is the amplitude, f_m the frequency, and φ_m the phase, of the m^{th} individual component.

Assuming random phases and summing the square of all amplitudes in a small range df of frequencies gives the wave frequency spectrum $S(f)$ of the wave signal:

$$\frac{1}{df}\sum_{f}^{f+df} \tfrac{1}{2}A_m^2 = S(f) \tag{2}$$

This can be computed as

$$S(f) = Z^* Z = |Z|^2 \tag{3}$$

from the Fourier transform

$$Z(f) = \int \Delta z(t) e^{-i2\pi f t}\mathrm{dt} \tag{4}$$

of the displacement signal.

Directional buoys can also measure horizontal displacements. Using the same sum of sinusoidal signals, these would be given as

$$\Delta x(t) = \sum_{m=1}^{M} A_m \cos\theta_m \sin[-2\pi f_m t + \phi_m] \tag{5}$$