DECISION COMPROMISE MODELLING BASED ON OWA OPERATORS

Eric LEVRAT[1] Jean RENAUD[2], Christian FONTEIX[3]

[1]CRAN UHP-INPL (Centre de Recherche d'Automatique de Nancy), UMR CNRS 7039
Boulevard des Aiguillettes 54506 Vandoeuvre Cedex, France
[2]ERPI (Equipe de Recherche des processus Innovatifs), EA 3767
8, rue Bastien Lepage, 54010 Nancy Cedex, France
[3]LSGC (Laboratoire des Sciences du Génie Chimique), UPR CNRS 6811
1, rue Gandville, BP451, 54001 Nancy Cedex, France
Phone: +03383684455, E-mail: eric.levrat@cran.uhp-nancy.fr

Abstract: This paper is concerned by the industrial application of Ordered Weighted Averaging OWA operators, introduced by Yager. These operators allow to express the type of compromises, by the notion of linguistic quantifiers, such as "most" of criteria. The interest of this method of aggregation is, beyond its simplicity of use, its evaluation of products according a unique scale. Furthermore, the weights are not fixed by criteria but by levels of performance. In this paper we present a methodology of classification of products by two approaches. The first one is based on a learning sample and the second one on linguistic quantifiers. An industrial application, from a food production, illustrates these approaches. We then discuss the classifications obtained by these two approaches and we present a comparison.

Keywords: multi-criteria analysis, subjective evaluation, OWA, linguistic quantifier, food industry

1. INTRODUCTION

In many industrial sectors such as food, cosmetic, medical and textile sectors, the manufacturer wants to launch on the market as soon as possible good products (Wu, *et al.*, 1997). These products are considered as the best "compromises" on all criteria according to a known target market and evolving in the time. Top management aims at a reduction of time to market for new products. As a result, a best "compromise" has to be defined according to the target market characteristics. This approach allows the manufacturer to list the innovative products and to exclude products with risk (worse on one or two criteria) or to retain atypical products (Bhaskar, *et al.*, 2001)

In this context, the decision-maker takes into account a multi-objective dimension during the design or manufacture of products. The human aspect becomes the central element of the resolution problem. A modelling of the preference of the decision-maker is necessary (Barthelemy, *et al.*, 2002).

Whatever the multicriteria decision problem, the choice of a solution requires the use of an aggregation operator, corresponding to the computation of a total satisfaction function based on local satisfactions for each criterion. Within the traditional framework of the multiattributes utility model, various unidimensional utility functions are aggregated into a single utility function combining all the criteria. In the case outranking methods, like Electre or Promethee, preference relations of pairs of alternatives are aggregated (Brans, *et al.*, 1984; Roy, and Bouyssou, 1993). Most of these methods or models usually make use of the comparisons of actions including criteria affected by values of different weights. However, the weighting stays a delicate operation, a criterion is more important than the other one in an objective or subjective way. This measure of differentiation is called weight of the criterion; it can be quantitative, qualitative, ordinal or cardinal. The valuation of weights is a subject which has been interesting the scientific community for these last decades (Roy, and Mousseau, 1996). All these methods aim at allocating a weight or an importance for a given set of criteria. For example, a product has to be "good" on such or such criterion. The industrial decision-maker privileges certain criteria with regard to their performances. If the industrial decision-maker wishes to privilege the performance of a profile of compromise and not a particular criterion, in order to obtain specific products, then weighting will not be allocated to a given criterion but to a given performance.

To solve this industrial problem, we have chosen a total aggregation method, Ordered Weight Average operators based on the classical weighted average (Yager, 1988; Grabish, 1996; Yager, 1996; Yager, 2003; Chakraborty, 2004). The principle of this method is that it allocates weighted factors indifferently to criteria, according to their performance, classified in decreasing order. The choice to use the OWA operator is justified for various reasons:

- products are classified according to the same scale, according to a profile of performance from the decision-maker (Xu, 2004),
- weights are used to express the level of performance, the nature of the compromise,
- OWA operators have a mechanism of calculation close to the weighted average.

Another interest, as Zadeh suggests, is the ablility to express linguistic quantifiers "rough", from fuzzy sets on a limited interval (Zadeh, 1975). These quantifiers indicate the degree of satisfaction of concepts such as: "at least some criteria must be satisfied", "most of criteria are satisfied" (Duboi,s and Prade, 1986), (Carlson, and Fuller, 2003).

The objective of this paper is to propose an industrial answer to the decision-maker during the design process of products by using OWA operators from linguistic quantifiers. In section 2, we present the OWA operators. In section 3, we present two approaches, the first one is based on learning process and the second one is supported by linguistic quantifiers. To confirm our methodology, we apply it to an industrial case from a collection of 47 food products valued by sensory analysis. In section 4, we present the learning process. Then, in section 5, three profiles of linguistic quantifiers are proposed before the application of the OWA operators. Section 6 presents a comparison of these two approaches and a discussion. Section 7 provides a conclusion.

2.OWA OPERATORS

Let us define a multicriteria decision making problem as composed of a set of potential solutions, and a set of properties or criteria X which represent an evaluation point of view of the solutions P. We wish to model the preferences of the decision maker concerning the potential solutions, in the form of a utility function depending on X and having the property:

$$x, y \quad P, \quad if\ x\ is\ prefered\ to\ y \quad then \\ x \succ y \quad u(x) > u(y) \tag{1}$$

A traditional way to build u is to incorporate several monodimensional utility functions using an operator F such as:

$$u(x) = F(u_1(x_1), \cdots, u_n(x_n)) \tag{2}$$

The most current aggregation tool is the weighted arithmetical mean, whose drawbacks are well-known. The fuzzy sets theory provides some useful tools to tackle these problems (Dubois, and Prade, 1986). In this paper, we are interested in the OWA, Ordered Weighted Average introduced by Yager (Yager, 1988) and defined as follows: let W be a weighting vector of dimension n with $W=[w_1\ w_2\ ...\ w_n]^T$ such that :

$$\sum_i w_i = 1 \quad ; \quad w_i \quad [0,1] \quad i \quad [1,n] \tag{3}$$

An OWA operator F of dimension n is defined as a mapping:

$$F : [0,1]^n \quad [0,1] \quad with \quad F(u_1, u_2, \cdots, u_n) = \sum_{j=1}^{n} w_j b_j ,$$

where b_j is the j^{th} greatest element of $\{u_1(x), ..., u_n(x)\}$. The weight w_i is not associated to a given criterion, but to the row which is taken by the utility value in the classification. According to W values, OWA operators include operators such as minimum, maximum, arithmetic mean and median value so that we obtain:

$$Min_i\ u_i(x) \quad OWA_i\ u_i(x) \quad Max_i\ u_i(x)$$

More generaly OWA operators satisfy the commutativity, monotonicity and idempotence properties and they are stable by linear transformation. OWA operators are members of a more generaly family of aggregation operators named Sugeno integrals and Choquet integrals (Grabish, 1996). Their main interest is that they can express directly the type of compromise or the compromise intensity wished by the decision maker by using vague quantifiers, such as: "at least some criteria must be taken into account". Thus, we can introduce a new behaviour of the aggregation function.

One difficulty consists in estimating the values of the weighting vector W for a given application. In this paper, we want to implement and to compare two approaches for the weights estimation. First, in (Filev, and Yager, 1998), the authors tackle the problem of the weights identification, in the case of the application of the OWA to a set of samples. Thus, starting from a set of samples, classified by an expert of the product, and whose global utility is evaluated or arbitrarily given, we want to determine the associated weights, in order to use these weights with the whole product set. In a second way, we exploit the concept of linguistic quantifier that allows the expert to moderate his search for compromise, in order to carry out a comparison. For the same objective, we show how linguistic quantifiers can be used in order to allow the expert to translate his preferences about decision into weighting vector. Both approaches are summarized in Fig. 1.

3. INDUSTRIAL APPLICATION

3.1 Experimental data

In order to carry out the comparison of the two approaches, a product data set has been collected. It consists of a three months production of agroalimentary products (corresponding to 47