GEOMETRIC DESIGN OF OBSERVERS FOR LINEAR TIME-DELAY SYSTEMS

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Abstract: The problem of designing an observer is considered for linear time delay systems with commensurable delays. The use of models over rings and of geometric methods allow us to develop a complete analysis of the problem and to describe constructive procedures for its solution.

Keywords: Systems over rings, Geometric approach, Unknown input observer

1. INTRODUCTION

Time delay is an inherent property of many physical systems and industrial applications, where delays are unavoidable effects of the transportation of materials or information through slow or very long communication lines. This motivated an increasing interest on analysis and synthesis techniques for time-delay systems in recent years, see, for instance, the Proceedings of the IFAC Workshops on Time Delay Systems in the years 2004 and 2005 and the references therein.
Several fundamental design procedures are based on the knowledge of the state of the system, but in most practical cases, either the states of the time-delay system are not physically available for direct measurement or the cost of the measurement is prohibitively high. Therefore the problem of state reconstruction for time delay systems, under various assumptions, has been investigated by several authors. An interesting case in a number of applications is the Unknown Input Observation Problem, namely the asymptotic state estimation in presence of unknown inputs, that often model disturbances.
The approach we follow makes use of models with coefficients in a ring that allow to exploit the power of geometric tools and methods for analyzing and solving design problems for systems with delays (see (Conte and Perdon, 1995*b*), (Conte and Perdon, 1995*a*), (Conte and Perdon, 2005)).
The standard approach to observer design for linear systems is based on the "Luenberger observer" where the measurement function is the identity. The generalization to systems over rings presents highly technical difficulties, as the extension of the pole-shifting theorem, or requires strong algebraic notions of observability. In (Hautus and Sontag, 1980) necessary and sufficient conditions for the existence of observers for systems over a ring of rational functions, using a transfer function approach, were given. In (Yao *et al.*, 1996), using a transfer function approach, a parametrization of observers for delay systems is given.
In this paper we discuss the existence of unknown inputs observers for systems over rings in the context of the geometric approach that provides a deep insight on the structural aspects and easy computable procedures (see (Perdon *et al.*, 2003)).

2. DELAY SYSTEMS MODELLED BY SYSTEMS OVER RINGS

Let $\mathbf{\Sigma}_d$ be a delay-differential system with a finite number of commensurable delays, described by the equations:

$$
\begin{cases}
\dot{x}(t) = \sum_{i=0}^{a} A_i x(t-ih) + \sum_{i=0}^{b} B_i u(t-ih) + \\
\qquad\qquad + \sum_{i=0}^{d} D_i q(t-ih) \\
y(t) = \sum_{i=0}^{c} C_i x(t-ih) \\
x(t) = \varphi(t),\ t \in [-\alpha\, h, 0] \quad \alpha > 0
\end{cases} \tag{1}
$$

where, denoting by $\mathbb{R}$ the field of real numbers, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, $q(t)$ represents an unknown input belonging to $\mathbb{R}^q$, $q \le p$, $\alpha = l.c.m.(a,b,c,d) \in \mathbb{N}$, $h \in \mathbb{R}^+$ is the delay, $\varphi(t)$ is the initial condition, A_i, B_i, C_i and D_i are matrices of suitable dimensions with entries in $\mathbb{R}$. By introducing the delay operator δ defined, for any time function $f(t)$, by $\delta^j f(t) := f(t-jh)$, the system (1) can be written as

$$
\Sigma_d \begin{cases}
\dot{x}(t) = \sum_{i=0}^{a} A_i \delta^i x(t) + \sum_{i=0}^{b} B_i \delta^i u(t) + \\
\qquad \sum_{i=0}^{d} D_i \delta^i q(t) \\
y(t) = \sum_{i=0}^{c} C_i \delta^i x(t).
\end{cases}
$$

By formally replacing the delay operator δ by the algebraic indeterminate Δ, it is possible to associate to $\mathbf{\Sigma}_d$ the system $\mathbf{\Sigma} = (A,\ B,\ C,\ D)$, defined by the equations

$$
\mathbf{\Sigma} \begin{cases}
x(t+1) = Ax(t) + Bu(t) + Dq(t) \\
y(t) = Cx(t)
\end{cases} \tag{2}
$$

where $A := \sum_{i=0}^{a} A_i \Delta^i$, $B := \sum_{i=0}^{b} B_i \Delta^i$, $C := \sum_{i=0}^{c} C_i \Delta^i$ and $D := \sum_{i=0}^{d} D_i \Delta^i$ are matrices with entries in the ring of real polynomials in one indeterminate, $\mathcal{R} = \mathbb{R}[\Delta]$. The state $x(t)$, the control input $u(t)$, the unknown input $q(t)$, the output $y(t)$ belong, respectively, to the free modules $\mathcal{X} = \mathcal{R}^n$, $\mathcal{U} = \mathcal{R}^m$, $\mathcal{Q} = \mathcal{R}^q$ and $\mathcal{Y} = \mathcal{R}^p$. $\mathbf{\Sigma}_d$ and $\mathbf{\Sigma}$ (2) are obviously different objects from a dynamical point of view, but they share the "structural" properties that depend only on the defining matrices $(A,\ B,\ C,\ D)$. Therefore many control problems concerning the input/output behavior of $\mathbf{\Sigma}_d$, can be naturally formulated in terms of the input/output behavior of $\mathbf{\Sigma}$ and solved in the framework of systems over rings. The solutions found in the system over rings framework can be interpreted in the original delay-differential setting.

Particular attention must be paid in dealing with the notion of stability and asymptotic convergence by means of systems with coefficients in a ring. A ring cannot, in general, be endowed with a natural metric structure and the distance of an element from zero cannot be defined, therefore we shall use a formal definition. Since we are using ring models to study delay systems, we will adopt the following definition.

Definition 1. The *Hurwitz polynomials* are polynomials in s with coefficients in $\mathcal{R}$ that belong to the set $\mathcal{H} = \{p(s,\Delta) \in \mathcal{R}[s] = \mathbb{R}[\Delta, s] \mid p$ monic and $p(\bar{s}, e^{-\bar{s}h}) \neq 0\ \forall\ \bar{s} \in \mathbb{C},\ Re(\bar{s}) \ge 0\}$.
A system (2) over the ring $\mathcal{R}$ is *(formally) stable* if $\det(sI - A) \in \mathcal{H}$.

3. PRELIMINARIES AND STATEMENT OF THE PROBLEM

Definition 2. A delay-differential system described by the equations:

$$
\begin{cases}
\dot{z}(t) = \sum_{i=0}^{a} \tilde{A}_i z(t-ih) + \sum_{i=0}^{b} \tilde{B}_i u(t-ih) + \\
\qquad\qquad + \sum_{i=0}^{g} \tilde{G}_i y(t-ih) \\
\hat{w}(t) = \sum_{i=0}^{l} L_i z(t-ih) + \sum_{i=0}^{k} K_i y(t-ih) \\
z(t) = \phi(t),\ t \in [-\beta\, h, 0] \quad \beta > 0
\end{cases} \tag{3}
$$

where $z(t) \in \mathbb{R}^s$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, $\beta = l.c.m.(a,b,g,l,k) \in \mathbb{N}$, $\phi(t)$ is the initial condition, $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{G}_i$, L_i, and K_i are matrices of suitable dimensions with entries in $\mathbb{R}$, is an *observer* for (1) if the output $\hat{w}(t)$ asymptotically converges to a linear function of the state $x(t)$ possibly with some delays, namely if $\lim_{t\to\infty}\left(\hat{w}(t) - \sum_{i=0}^{\bar{h}} H_i x(t-ih)\right) = 0$. Denoting by H the matrix $H := \sum_{i=0}^{\bar{h}} H_i \delta^i$, we shall write, shortly, that

$$
\lim_{t\to\infty} (\hat{w}(t) - Hx(t)) = 0. \tag{4}
$$

When H is the identity matrix (3) is called an *identity* observer.

Let Σ be the finite-dimensional linear time-invariant dynamical system over the ring $\mathcal{R} = \mathbb{R}[\Delta]$ associated to system (1), given by the equations (2). We will assume that $p \ge q$ and, without loss of generality, that $rank(C) = p$ and $rank(D) = q$. Due to the lack of information on the disturbance $q(t)$, often it is not possible to reconstruct the entire state module of Σ.

Definition 3. An r-dimensional linear system Σ_O of the form:

$$
\Sigma_O : \begin{cases}
z(t+1) = \tilde{A}z(t) + \tilde{B}u(t) + \tilde{G}y(t) \\
\hat{w}(t) = Lz(t) + Ky(t)
\end{cases} \tag{5}
$$

whose output $\hat{w}(t)$, *independently from* $q(t)$, asymptotically converges to $Hx(t)$, with $H \in \mathcal{R}^{s\times n}$, is an *unknown-input observer (U.I.O.)* for Σ. Σ_O is called a *full-order* (*reduced-order*) observer if $r = n$ ($r < n$).