NONLINEAR OUTPUT REGULATION AND CROSS-TERMS COMPENSATION [1]

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Abstract: In this paper we survey the results recently obtained in (Marconi *et al.*, 2006) in the context of output regulation and we frame them in a broader context of set stabilization by output feedback. The peculiarity of the set stabilization framework which is here formulated is given by the presence of non-vanishing cross-terms between zero- and output-dynamics. We show how the mathematical tools developed in (Marconi *et al.*, 2006) allow for the synthesis of a robust control law which is not inspired by certainty equivalence principles. As new application of the tools we show how a problem of output feedback asymptotic stabilization with zero dynamics which are not exponentially stable can be handled by means of high-gain output feedback.

Keywords: Output regulation, robust asymptotic stabilization, internal model principle, robust control.

1. INTRODUCTION

A noteworthy research attempt within the nonlinear control theory have been recently devoted to the problem of output regulation in which a number of tracking, disturbance suppression and robust asymptotic stabilization problems can be framed (see (Byrnes and Isidori, 2003)). The peculiarity of the problem of output regulation relies in the fact that the controlled plant is affected by exogenous variables (representing references to be tracked and/or disturbances to be rejected and/or parametric uncertainties), generated by an autonomous system (the so-called "exo-system"), whose effect has to be asymptotically counteracted by a suitable design of the controller. As pioneered in a linear setting in (Francis and Wonham, 1976) and in a nonlinear setting in (Isidori and Byrnes, 1990), the controller solving the problem at hand has to be designed by employing the a-priori knowledge of the environment in which the plant operates provided, in the classical framework, by the structure of the exo-system. This, in turn, has led to the celebrated concept of *internal model* and to the identification of design procedures for *internal model-based regulators.* In this context the crucial property required to any regulator solving the problem is to be able to generate all possible "feed-forward inputs" which force an identically zero regulation error, namely the control input able to render invariant the so-called zero error manifold. Such a control input is what, in the classical terminology, is called *the friend.*

An important distinguishing feature which characterizes the problem of output regulation and any meaningful solution to it with respect to others tracking/disturbance rejection frameworks, is the ability to design internal model-based regulators without relying upon *certainty equivalence principles* in the friend generation. In others words

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the solution, to be really effective, is required to generate the desired steady-state control law without an explicit estimation of the internal dynamics and, in particular, of the exo-system state variables. This feature acquires a special importance and meaning in presence of uncertain parameters in the controlled plant which render, in general, ineffective solutions based on explicit estimations and "exalt" internal model-based design methodologies. To think of conventional set point control in a linear setting, to this respect, is instructive (see (Francis and Wonham, 1976)).

In the last ten years or so, the related nonlinear literature has witnessed a number of works aiming to identify even more general procedures to design internal models. In particular the attempt was directed to weaken even more the so-called "immersion assumption", requiring that the dynamical system defining all possible "feed-forward inputs" which force an identically zero regulation error were "immersed" into a system exhibiting certain structural properties, by providing a steady development of less stringent assumptions: immersion into a linear *known* observable system (see (Huang and Lin, 1994), (Khalil, 1994), (Serrani *et al.*, 2000)), immersion into a linear *un-known* (but linearly parameterized) system ((Serrani *et al.*, 2001)), immersion into a linear system having a nonlinear output map ((Chen and Huang, 2004)), immersion into a system in canonical observability form ((Byrnes and Isidori, 2004)), immersion into a system in a nonlinear adaptive observability form ((DelliPriscoli *et al.*, 2006*b*)), are only a few examples testifying the richness and liveliness of the past literature on this topic.
As highlighted in (DelliPriscoli *et al.*, 2006*a*) (see also (Marconi and Isidori, 2007)), the common idea in the previous works was to draw inspiration from typical design methodologies of linear as well as nonlinear observers in order to design internal models. This perspective, along with the new methodology to design nonlinear observers proposed in (Andrieu and Praly, 2006), was the primary source of inspiration used in (Marconi *et al.*, 2006) to completely weaken the immersion assumption. In that work the key result has been precisely to find a design methodology to make invariant a compact attractor (the zero error manifold in the output regulation context) by output feedback for a special class of nonlinear systems without any specific requirement on the associated friend. This, in turn, has led to a theory of output regulation without immersion.

The main aim of this paper is to survey the basic mathematical tools proposed in (Marconi *et al.*, 2006) by showing their usefulness in a broader context involving a particular set stabilization problem for nonlinear systems. The peculiarity of the set stabilization framework which is here formulated is given by the presence of non-vanishing cross-terms between zero- and output-dynamics which render ineffective classical high-gain stabilization techniques and ask for more sophisticated control law based on the internal model. As a new application of the presented ideas, we show how it is possible to weaken the requirement of exponential stability of the zero dynamics in the problem of stabilizing by high-gain output feedback a particular class of minimum-phase nonlinear systems.

The paper is, in some parts, deliberately not technical as the main goal is to provide the main ideas and possible further (beyond output regulation) exploitations of the paper (Marconi *et al.*, 2006) where the interested reader can find all the technical details. A nice complement to this work is provided by the paper (Marconi and Praly, 2007) in which the ideas presented here in section 4 are generalized in the context of stabilization of nonlinear systems via nonlinear separation principle.

2. THE GENERAL FRAMEWORK

2.1 A generalized problem of output feedback stabilization

A remarkable number of the works which, in the past literature, addressed problems of robust output feedback stabilization have been focused on the class of smooth systems described as

$$\begin{aligned} \dot{x} &= f(x,y) & x \in \mathbb{R}^n\,, \\ \dot{y} &= q(x,y) + b(x,y)u & y,u \in \mathbb{R} \end{aligned} \tag{1}$$

with measured output y and control input u. This class of systems, indeed, is in the well-known and celebrated normal form (see (Isidori, 1999)) with unitary relative degree [2] and *high-frequency gain* $b(x,y)$ (with $b(x,y) \neq 0$ for any (x,y) in the domain of interest).

In this setting a possible meaningful control problem is the one which asks, for a given compact set $\mathcal{B} \in \mathbb{R}^n$ which is forward invariant for the *zero dynamics* $\dot{x} = f(x,0)$, to design an output (y) feedback controller so that the set $\mathcal{B} \times \{0\}$ is locally asymptotically stable with a suitable domain of attraction. Clearly, in the particular case in which $\mathcal{B}$ coincides with a "simple" equilibrium point of the zero dynamics, the problem reduces to a conventional stabilization problem for which a number of works have been devoted (see (Teel and Praly, 1995) and the reference therein). On the other hand, in the general case, the problem at hand can be cast as a particular set stabilization problem in which the goal is to steer to zero only a

[2] All the forthcoming reasonings can be generalized to the higher relative degree case without any conceptual added value.