A NON-EQUILIBRIUM ANALYSIS & CONTROL FRAMEWORK FOR COMMUNICATION NETWORKS

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Abstract: We present a non-equilibrium analysis and control approach for the Active Queue Management (AQM) problem in communication networks. Using simplified fluid models we carry out a bifurcation study of the complex dynamic queue behavior to show that non-equilibrium methods are essential for analysis and optimization in the AQM problem. We investigate an ergodic theoretic framework for stochastic modeling of the non-equilibrium behavior in deterministic models and use it to identify parameters of a fluid model from packet level simulations. For computational tractability, we use set-oriented numerical methods to construct finite-dimensional Markov models. Subsequently, we develop and analyze an example AQM algorithm using a Markov Decision Process (MDP) based control framework. The control scheme developed is optimal with respect to a reward function defined over the queue size and aggregate flow rate. We implement and simulate our illustrative AQM algorithm in the *ns-2* network simulator. The initial results obtained confirm the theoretical analysis and exhibit promising performance when compared with well-known alternative schemes under persistent non-equilibrium queue behavior.

Keywords: Communication networks, active queue management, Markov models, optimal control.

1. INTRODUCTION

Communication networks such as the Internet exhibit a wide variety of complex *non-equilibrium* dynamic behavior. Examples of such behavior include self-excited user flow rate oscillations in the presence of delays, dynamic synchronization of the flows passing through the same bottleneck link, and chaotic behavior of user flows and queues at the routers. However, the optimization and control methods for such networks are primarily based upon an equilibrium framework. We note the Kelly's framework (Kelly et al., 1998) or game theoretic framework (Alpcan and Başar, 2005) and refer to (Srikant, 2004) for an overview of the literature on the subject.

This paper is concerned with methods for analysis and control of non-equilibrium behavior in communication networks. Specifically, we consider the Active Queue Management (AQM) problem. AQM provides a mechanism by which a router sends advanced congestion notification to the users in order to regulate the flow rates. This can be accomplished either by dropping packets or marking them. Since the earliest Droptail scheme, several AQM schemes have been proposed and studied in literature including RED, E-RED, REM, AVQ, and BLUE (Srikant, 2004).

We are primarily motivated by the analysis and control of non-equilibrium queue behavior which arises primarily as a result of nonlinear dynamics. There is a gap between available methods that focus on static optimization, and simulations that show persistent non-equilibrium behavior that does not need any noise. From a practical viewpoint, explicit analysis and control of non-equilibrium behavior could play an important role in the performance of the overall congestion control scheme.

In this paper, we represent for modeling as well as control the dynamic variables by their stochastic counterparts. Even though the models are deterministic the analysis and control approach is stochastic. The modeling approach is based upon the methods of Ergodic theory for representing complex behavior in nonlinear dynamical systems. In particular, we replace the dynamical models by their stochastic counterparts - the so-called Perron-Frobenius operator (Lasota and Mackey, 1994) While, the dynamical model propagates the initial condition, the Perron-Frobenius (P-F) operator propagates uncertainty in initial condition. The main advantage of this approach is the often easier represention of complex asymptotic dynamic behavior as invariant probability measures of the P-F operator.

We use the recent set-oriented numerical methods for discretization of the dynamical systems (Dellnitz and Junge, 2002; Froyland, 2001). Our goal is to use these simulation based methods to construct finite-dimensional Markov chains from the dynamic model. These Markov chains are then used for both the computational bifurcation analysis as well as for control design. The stochastic modeling approach we take enables us to carry out a *bifurcation analysis* to understand qualitative changes in queue behavior, *identification* of the fluid model parameters from packet level *ns-2* simulations, and *control synthesis* for shaping non-equilibrium behavior.

The outline for the rest of the paper is as follows: In Section 2, a well-known network model of user and queue behavior is presented. Next, a stochastic modeling of the network model together with its discrete approximation as finite-dimensional Markov models is described and used to carry out bifurcation analysis as well as identification of network model parameters using the ns-2 simulations. In Section 4, an MDP-based framework for optimization and control of these models is summarized. The ns-2 simulation results demonstrating control of non-equilibrium behavior under the AQM algorithm developed are described in Section 5. The paper ends with the concluding remarks of Section 6.

2. DETERMINISTIC FLUID MODEL

2.1 Single Bottleneck Link with Symmetric Users

We consider a single bottleneck link of a network with fixed capacity C shared by M users. Instead of conducting a packet level analysis of the network we adopt a network model based on fluid approximations (Srikant, 2004; Alpcan and Başar, 2005). Each user is associated with an unique connection for simplicity and transmits data with a nonnegative flow rate $x_i \in \mathbb{R}^+ \doteq [0, \infty)$ over this bottleneck link. The i^{th} user is assumed to follow a transfer control protocol (TCP)-like additive-increase multiplicative-decrease flow control scheme,

$$\dot{x}_i(t) = \kappa \left(\frac{1}{d} - \beta x_i(t)^2 p(t) \right), \qquad (1)$$

where $0 \leq p \leq 1$ is the observed rate of marking (or depending on the implementation, dropping) of its packets, κ denotes the step-size, and d and β denote the (symmetric) rate-increase and decrease parameters, respectively. For a prescribed $p(t)$, the ODE (1) has a well-defined solution in $\mathbb{R}^+$ for all time because the right hand side is Lipschitz in x_i and $\mathbb{R}^+$ is a positively invariant set with respect to (1).

The packet marking occurs at the link whose dynamics are next described. If the aggregate sending rate of users exceeds the capacity C of the link, then the arriving packets are queued in the buffer q of the link. The non-negative queue size evolves according to the ODE

$$\dot{q}(t) = \begin{cases} \sum_{i=1}^{M} x_i(t) - C & q \in (0, B), \\ \min(0, \sum_{i=1}^{M} x_i(t) - C) & q = B, \\ \max(0, \sum_{i=1}^{M} x_i(t) - C) & q = 0. \end{cases} \qquad (2)$$

where we assume a maximum buffer size of B at which the queue saturates and any incoming packet after this point is dropped; cf. (Hollot et al., 2002). The function $p(\cdot)$ in (1) is set by the AQM control and takes the general form $p = F(q)$. As an example, packet marking for the widely used droptail AQM scheme is described by

$$p = \begin{cases} 0 & \text{, if } q < B \\ 1 & \text{, otherwise .} \end{cases} \qquad (3)$$

With a large number of users M, a detailed non-equilibrium analysis of the multi-user model (1) is infeasible. In order to simplify the analysis, we note that the equations are equivariant with respect to the permutation group with the group action $x_i \to x_j$ for all $i, j \in \{1, \ldots, M\}$. As a result of this symmetry, the linear subspace $S = \{\underline{x} \in \mathbb{R}^{+M} : x_i = x_1\}$ is a fixed-point space, which we will also refer to as the synchrony subspace. In particular, the subspace S is positively invariant with respect to dynamics of (1).