A STATE FEEDBACK CONTROL METHODOLOGY FOR NONLINEAR SYSTEMS WITH TIME/STATE DEPENDENT INPUT DELAY

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Abstract: This paper proposes a methodology for state feedback stabilization of nonlinear systems with time-varying input delay, and more particularly when the delay varies w.r.t. the state variables. The control design approach is based on state prediction computation, and the numerical issue resulting from the actual implementation of such a control law is also discussed. The overall method is in particular illustrated on an example of water flow control in open-channel systems.

Keywords: Nonlinear control, nonlinear time-delay systems, time-varying delay, state-dependent delay, state feedback, state predictor.

1. INTRODUCTION

Time-delay systems belong to the class of infinite-dimensional systems. In the linear case, a time-delay system has in general an infinite number of eigenvalues. Control laws have been proposed to assign a finite number of eigenvalues in closed loop (Manitius and Olbrot [1979]). This approach is called the finite spectrum assignment problem. Solutions to this problem are obtained in terms of delay-distributed control laws. However, the implementation of delay-distributed control laws is difficult due to the integral term which cannot be computed explicitly. In (Manitius and Olbrot [1979]), it is suggested to approximate the integral by a sum of point-wise delays by using a quadrature rule. However, this approach may fail due to the occurrence of unstable poles introduced by the discretization procedure (Assche et al. [1999]). The use of block-pulse functions has also been proposed in (Fattouh et al. [2001]). More recently, a safe implementation of delay-distributed control laws has been proposed by using a low-pass filter in the control loop (Mondie and Michiels [2003]). In (Maza-Casas et al. [2000]) a passivity-based control scheme is proposed for the stabilization of SISO nonlinear systems with input delay. However, distributed-delay control laws for nonlinear systems has not yet been extensively studied. Other approaches have been proposed for special cases (Mazenc et al. [2003], Mazenc and Bliman [2006], Zhang et al. [2006]).
In the present paper, the purpose is to extend the so-called finite spectrum assignment approach already available for linear input-delayed systems, to the case of *nonlinear* input-delayed systems, with even *state-dependent* input delays. Notice that the stability problem for such systems with state-dependent delays was previously considered in Verriest [2002], but for linear systems, and that the work we propose here comes as a continuation of Georges et al. [2007] where the problem was already dedicated to nonlinear systems, but basi-

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cally with *constant* input delays.
The paper in organized as follows: section 2 first presents the formal statement of the proposed approach. Section 3 then discusses numerical issues for practical implementation, while section 4 proposes a possible use in water flow control as an illustrative example of application. Section 5 finally gives some conclusions.

2. A STABILIZATION SCHEME FOR NONLINEAR SYSTEMS WITH VARYING INPUT DELAY

Let us consider nonlinear systems with a varying input delay of the following general form:

$$\dot{x}(t) = F(x(t), u(t - \tau(t, x(t)))) \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $\tau(t, x(t))$ is a varying delay with known smooth evolution w.r.t. its arguments. The origin of the system is supposed to be an equilibrium point ($F(0, 0) = 0$).
Clearly, for causality, $\tau(t, x(t))$ should remain larger than 0 for any time and realizable trajectory, and in particular around the origin $x = 0$.
The purpose here is to design a state feedback law in order to stabilize the origin of the system in closed loop. To that end, the idea is to extend the so-called finite spectrum assignment approach already available for linear input-delayed systems (Mondie and Michiels [2003]).

This approach is based on the following principle: firstly a prediction of the state at an appropriate prediction time δ, denoted by $x_p(t, t + \delta)$, is computed from the available state $x(t)$ at time t and input controls $u(\theta)$, $\theta \in [t - \delta, t]$. Then the predicted state is used to compute the control law. The prediction time is chosen so that the effect of the delay vanishes and the closed-loop system is no more a time-delay system.
In the case of *constant delay*, the prediction time is just given by the delay τ itself (e.g. as in Georges et al. [2007]). When the delay might be varying $\tau(t)$, the prediction horizon cannot be τ anymore but a time-varying prediction horizon $\delta(t)$ chosen such that $\delta(t) = \tau(t + \delta(t))$. This condition is used to ensure that a control input $u(t)$ can be computed since in this case one has $u(t - \tau(t + \delta(t)) + \delta(t)) = u(t)$(Witrant et al. [2004]).
The same obviously occurs when the delay further depends on the state: δ is to be chosen such that $\delta(t) = \tau(t + \delta(t), x(t + \delta(t)))$.
The stability of the closed-loop system

$$\dot{x}(t + \delta(t)) = F(x(t + \delta(t)), \Phi(x(t + \delta(t)))) \quad (2)$$

expressed in the time coordinate $t + \delta(t)$, can then be guaranteed if there exists a smooth feedback $u(t) = \Phi(x(t))$ ensuring the closed-loop stability of the non-delayed system $\dot{x}(t) = F(x(t), u(t))$.

The main issue remains the computation at time t of the prediction of the state at time $t + \delta(t)$, denoted by $x_p(t, t + \delta(t))$, which is given by:

$$x_p(t, t + \delta(t)) = x(t) + \\ + \int_t^{t+\delta(t)} F(x_p(t, \theta), u(\theta - \tau(\theta, x_p(t, \theta))))d\theta.$$

The predicted state $x_p(t, t + \delta(t))$ may also be defined in terms of an operator Ψ defined as follows:

$$x_p(t, t + \delta(t)) = \Psi(x(t), \{u(\theta)\}_{\theta \in [t-\tau, t]}) \qquad (3)$$

Finally, the control law is given by

$$u(t) = \Phi(\Psi(x(t), \{u(\theta)\}_{\theta \in [t-\tau, t]})) \qquad (4)$$

From now on, in order to simplify a little bit the notations, but still keeping the specificity of a state-dependent varying delay, we will limit the presentation to the case when $\tau = \tau(x(t))$ (without restriction).
The main result in that respect can then be stated as follows:

Theorem 1. If there exists a smooth state feedback $\Phi(x)$ making the origin of $\dot{x}(t) = F(x(t), \Phi(x(t)))$ locally exponentially stable - in the sense that (Khalil [1996]):
there exists $D \subset R^n$ containing and a $\mathcal{C}^1$ positive definite function $V : D \to R$ such that $\forall x \in D$:

(i) $c_1\|x\|^2 \leq V(x) \leq c_2\|x\|^2$

(ii) $\dfrac{\partial V(x)}{\partial x} F(x, \Phi(x)) \leq -c_3\|x\|^2$

for positive constants c_1, c_2 and c_3,
then the control law (4) makes the origin of (1) locally asymptotically stable.

Proof: First of all, define $f(x) := F(x, \Phi(x))$ and $z(t) := x(t + \delta(t))$. Then the closed-loop system (1)-(4) can be re-written w.r.t. z and f as:

$$\dot{z}(t) = (1 + \dot{\delta}(t))f(z(t)). \qquad (5)$$

Let us then consider $V(z)$ as a candidate Lyapunov function for this system. Clearly:

$$\dot{V} = (1 + \dot{\delta}(t))\frac{\partial V}{\partial x} f(z(t)).$$

Now notice that from the definition of δ we have:

$$[1 - \frac{\partial \tau}{\partial x}(z)f(z)]\dot{\delta} = \frac{\partial \tau}{\partial x}(z)f(z) \qquad (6)$$

where $\frac{\partial \tau}{\partial x}(z)f(z)$ vanishes when z goes to zero.
Hence there exists a domain $\bar{D} \subset \mathbb{R}^n$ of states z of D and containing the origin, such that $1 - \frac{\partial \tau}{\partial x}(z)f(z) > 0$, and consequently such that $\dot{\delta} >$