STATE ESTIMATION FOR TWO OUTPUT SYSTEMS IN DISCRETE TIME

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Abstract: The paper deals with the equivalence under coordinates change and output transformation to an observer canonical form for discrete–time systems with two outputs. Necessary and sufficient conditions for local equivalence are given.
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1. INTRODUCTION

In the present paper we address the problem of the equivalence under coordinates change and output transformation to an observer canonical form for discrete time systems with two output functions. The problem of the equivalence to observer canonical forms has been largely investigated both in continuous and discrete time (see the extensive literature on the argument, between the others: [Krener et al. , 1983], [Krener et al. , 1985], [Xia et al. , 1989], [Chung et al., 1990], [W. Lee et al. , 1991], [La Scala et al. , 1995], [Song et al. , 1995], [Moraal et al. , 1995], [Lin et al. , 1995], [Besançon et al., 1999], [Huijeberts , 1999], [Kazantzis et al. , 2001], [Krener et al. , 2002b], [Xiao , 2006]).

In the present paper we follow the geometric approach proposed in [Monaco et al. , 1997a] and already used with respect to the equivalence of observer canonical forms in [Califano et al., 2003] for nonautonomous single output discrete time systems and in [Califano et al., 2005] for multi output discrete time systems. The main result here proposed consists in necessary and sufficient conditions for the solvability of the problem with reference to the two output case. These conditions generalize the results proposed in [Hou et al., 1999] for continuous time systems. Work is in progress on the general case. We will thus consider a nonlinear discrete-time system

$$\begin{aligned} x(k+1) &= F(x(k), u(k)) \\ y(k) &= h(x(k)) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^2$, $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and h are analytic functions, $(0,0)$ is an equilibrium pair, i.e. $F(0,0) = 0$, and $h(0) = 0$. We will assume that the Jacobian matrix $JF_0(x)$ of $F_0(x)$ has full rank at $x_0 = 0$. Consequently, the drift term F_0 is locally invertible, as well as the the controlled dynamics $F(\cdot, u)$ locally in a neighborhood of $(0,0)$. $\mathcal{X}_0$ and $\mathcal{U}_0$ will denote such suitable neighborhoods of $x_0 = 0$ and $u = 0$ resp.

Definition 1. The observability index k_i, associated with the output h_i is the first integer s.t.

$$d(h_i \circ F_0^{k_i}) \in \text{span}\{dh, \cdots d(h \circ F_0^{k_i - 1})\} \tag{2}$$

while for $1 \le j \le k_i - 1$,

$$d(h_i \circ F_0^{j}) \notin \text{span}\{dh, \cdots d(h \circ F_0^{j - 1})\}. \tag{3}$$

Definition 2. The problem of the equivalence to the generalized observer form with output transformation has a solution if there exist a locally defined coordinates change $z = \phi(x)$ and an output transformation $\bar{y} = \varphi(y)$ s.t. in the new variables, (1) reads

$$
\begin{aligned}
z(k+1) &= A(u(k))z(k) + \Psi(y(k), u(k)) \\
\bar{y}(k) &= \varphi(y(k)) = Cz(k)
\end{aligned}
\tag{4}
$$

with $(A(0), C)$ an observable pair in the canonical Brunovskij form, i.e.

$$
A(0) = diag(A_1(0), A_2(0)),\;\; C = diag(C_1, C_2),
$$

$$
A_i(0) = \begin{pmatrix} 0 & \cdots & \cdots & \cdots & 0 \\ 1 & \ddots & & & \vdots \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix}_{k_i \times k_i} .
$$

$$
C_i = (\,0 \quad \cdots \quad 0 \quad 1\,)_{1 \times k_i}
$$

k_i is the observability index associated with $\bar{y}_i$.

Dropping the term "generalized " will mean that $A(u) = A(0) = A$, while dropping the term "with output transformation" will mean that $\bar{y} = y$.

The paper is organized as follows. Technical issues concerning the geometric framework are given in Section 2. In Section 3, the problem of local equivalence under coordinates change to the generalized output injection observer form for systems with two outputs is recalled. Necessary and sufficient conditions for the problem solvability are given in Section 4.

2. RECALLS AND NOTATIONS

The following notations are issued from [Monaco et al. , 1986], [Sontag , 1986], [Jakubczyk et al. , 1990]. Given two vector fields $\tau_1(x), \tau_2(x)$, a real valued function $\lambda(x)$ and a diffeomorphism $\phi(x)$, defined on $\mathbb{R}^n$, $L_{\tau_1}\lambda(x) := \dfrac{\partial \lambda(x)}{\partial x}\tau_1(x)$, is the standard Lie derivative, $ad_{\tau_1}\tau_2(x) := [\tau_1, \tau_2](x) = L_{\tau_1} \circ L_{\tau_2}(Id)|_x - L_{\tau_2} \circ L_{\tau_1}(Id)|_x$ is the Lie bracket of vector fields and $Ad_\phi \tau_1$ is the transport of τ_1 along $\phi(x)$, i.e. $Ad_\phi \tau_1 := \left(\dfrac{\partial \phi}{\partial x}\tau_1\right)\Big|_{\phi^{-1}}$. δ_{rs} denotes the kronecker index which is 0 for $r \neq s$ and 1 for $r = s$, Id denotes the identity function, I the identity operator and $J\phi(x) := \dfrac{\partial \phi(x)}{\partial x}$, the jacobian of $\phi(x)$.

Consider now ([Monaco et al. , 1997a], [Monaco et al. , 2007]) the parameterized family of vector fields associated with (1)

$$
{}_{i_1}G^0(x,u) \doteq \left[\frac{\partial}{\partial \varepsilon}\Big|_{\varepsilon=0} F(\cdot, u_1, \cdots, u_{i_1} + \varepsilon, \cdots, u_m)\right]_{F^{-1}(x,u)},
$$

$i_1 = 1, \cdots, m$, locally well defined in $\mathcal{U}_0$ and set for $i_1 \in [1,m]$, ${}_{i_1}G^0_1(x) \doteq {}_{i_1}G^0(x,0)$ and for $i_j \in [1,m]$, $j = 1, \cdots, k$, $k > 1$

$$
{}_{i_1,i_2\cdots i_k}G^0_k(x) \doteq \frac{\partial^{k-1}}{\partial u_{i_2}\cdots \partial u_{i_k}}\Big|_{u=0} {}_{i_1}G^0(x,u).
$$

Accordingly (1) admits the following exponential representation

$$
F(x,u) = F_0(x) + \sum_{i=1}^{m} u^i L_{{}_iG^0_1(\cdot)}(Id)|_{F_0(x)} +
$$

$$
+ \sum_{i,j=1}^{m} \frac{u^i u^j}{2}\left(L_{{}_{ij}G^0_2(\cdot)} + L_{{}_iG^0_1(\cdot)} \circ L_{{}_jG^0_1(\cdot)}\right)(Id)|_{F_0(x)} + 0(u^3)
$$

$$
= e^{u\mathcal{G}^0(\cdot,u)}(Id)\Big|_{F_0(x)} = \left(I + \sum_{p>0}\frac{1}{p!}L^p_{u\mathcal{G}^0(\cdot,u)}\right)(Id)\Big|_{F_0(x)} .
$$

$u\mathcal{G}^0(.,u) := \mathbb{R}^n \to \mathbb{R}^n$, is defined by its series expansion with respect to u. It is a smooth vector field parameterized by $(u_1, \cdots, u_m)$ and a Lie element in the $({}_{i_1\ldots i_k}G^0_k)$'s [Monaco et al. , 1997a].

The transport of a vector field τ_0 along the dynamics (1) is thus a vector field $\tau_1(\cdot, u)$ given by

$$
\tau_1(\cdot,u) = \left(\frac{\partial F(\cdot,u)}{\partial x}\tau_0(\cdot)\right)_{F^{-1}(\cdot,u)} = e^{-ad_{u\mathcal{G}^0(\cdot,u)}}(Ad_{F_0}\tau_0(\cdot))
$$

$$
= (I + \sum_{p\geq 1}\frac{(-1)^p}{p!}ad^p_{u\mathcal{G}^0(\cdot,u)})(Ad_{F_0}\tau_0(\cdot))
$$

$$
= Ad_{F_0}\tau_0(\cdot) - \sum_{i=1}^{m} u^i\; ad_{{}_iG^0_1}Ad_{F_0}\tau_0(\cdot) +
$$

$$
\sum_{i,j=1}^{m}\frac{u^i u^j}{2}(-ad_{{}_{ij}G^0_2} + ad_{{}_iG^0_1}ad_{{}_jG^0_1})Ad_{F_0}\tau_0(\cdot) + O(u^3)
$$

In the coordinates $z = \phi(x)$, we will have that ${}_i\tilde{G}^0(z,u) := Ad_\phi {}_iG^0(x,u)$, ${}_\eta\tilde{G}^0_k(z) := Ad_\phi\, {}_\eta G^0_k(x)$, $\forall k \geq 1$, $\eta = i_1, \cdots i_k$.

In the multi ouput case, a first problem concerns the introduction of a suitable set of observability indices. From (2–3) the observability indices can be computed from the codistributions

$$
\Omega_i = \operatorname{span}\{dh \cdots d(h \circ F^i_0)\}, \quad i \geq 0.
$$

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