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A finite nonlinear hyper-viscoelastic model for soft biological tissues

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ABSTRACT

Soft tissues exhibit highly nonlinear rate and time-dependent stress-strain behaviour. Strain and strain rate dependencies are often modelled using a hyperelastic model and a discrete (standard linear solid) or continuous spectrum (quasi-linear) viscoelastic model, respectively. However, these models are unable to properly capture the materials characteristics because hyperelastic models are unsuited for time-dependent events, whereas the common viscoelastic models are insufficient for the nonlinear and finite strain viscoelastic tissue responses. The convolution integral based models can demonstrate a finite viscoelastic response; however, their derivations are not consistent with the laws of thermodynamics. The aim of this work was to develop a three-dimensional finite hyper-viscoelastic model for soft tissues using a thermodynamically consistent approach. In addition, a nonlinear function, dependent on strain and strain rate, was adopted to capture the nonlinear variation of viscosity during a loading process. To demonstrate the efficacy and versatility of this approach, the model was used to recreate the experimental results performed on different types of soft tissues. In all the cases, the simulation results were well matched ($R^2 \geqslant 0.99$) with the experimental data.

1. Introduction

In the field of biomechanics, soft tissue characterisation through mathematical models has progressed rapidly. Biological tissues exhibit a highly nonlinear stress-strain relationship, in which the stress may depend on the strain, strain rate, and strain history (Holzapfel, 2000; Sanjeevi, 1982; Zhao et al., 2008).

To address such complex rate and time-dependent material responses, a viscoelastic modelling framework is necessary. The discrete element based standard linear solid (SLS) model or the continuous relaxation spectrum based quasi-linear viscoelastic model are traditionally the most popular choices for soft tissue modelling (Fung, 2013; Iatridis et al., 2003; Smith et al., 2005). However, the usage of these models is confined because of their inability to address the nonlinear and finite strain viscoelastic properties (Provenzano et al., 2002). Chung and Buist (2012) extended the SLS model capabilities to address the nonlinear soft tissue phenomena, but their model lacks the ability to deal with the true three-dimensional (3D) nature of the tissue.

Finite strain viscoelasticity is usually modelled using a different framework which is an extension of the classical Boltzmann superposition principle to finite strain. In this framework, the material responses are assumed to be the sum of strain rate independent and dependent behaviour where the strain rate dependency is modelled through a time convolution integral (Petiteau et al., 2013). A wealth of literature is available about models based on the convolution integral method (CIM) where each model utilises different fading memory functions to reflect the effect of strain history on stress (Miller and Chinzei, 1997; Miller, 1999; Troyer et al., 2012a,b; Troyer and Puttlitz, 2012). However, a major limitation of these models is that they are not founded on the principles of thermodynamics. Thus, the constitutive parameters must be chosen carefully and the validation of the second law of thermodynamics must be checked a posteriori for these models (Pioletti et al., 1998; Provenzano et al., 2001). In this context, Pioletti and Rakotomanana (2000) proposed a thermodynamically consistent CIM based model by assigning two different strain energy functions to the elastic and viscous properties of the material. However, their model needs a complex process to calibrate the relaxation time constants and parameters (Khajehsaeid et al., 2014).

Several works have demonstrated the existence of different stress descent rates in a single stress relaxation path, which corroborates the nonlinearity and strain-level dependency of viscosity (Van Loocke et al., 2009; Wheatley et al., 2016; Zhao et al., 2003). From a modelling point of view, this behaviour is usually described by introducing more than one dashpot element with different time constants and strain-dependent parameters. However, this leads to a diverse array of configurations even for the same tissue sample as

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the authors usually combine the dashpot elements in different ways to make them suitable for their experimental observations. Hence, a unique viscoelastic model consistent with the laws of thermodynamics and with capabilities to address finite deformations, nonlinear viscosity and 3D implementation in a compact form simultaneously is needed.

2. Materials and methods

Here, an isothermal deformation of soft tissues is considered. Letters $a, \boldsymbol{a}$, and $\mathbf{A}$ represent a scalar, a vector, and a second order tensor, respectively. In particular, $\mathbf{I}$ is the second order identity tensor, and $\mathbf{A}^T$ represents the transpose of a tensor. The trace and deviatoric part of a second order tensor are $\text{tr}(\mathbf{A}) = \mathbf{I} : \mathbf{A}$ and $\mathbf{A}^D = \mathbf{A} - (1/3)\text{tr}(\mathbf{A})\mathbf{I}$, where the $(:)$ represents the tensor scalar product. The magnitude of a tensor is denoted by $\|\mathbf{A}\| = \sqrt{\mathbf{A} : \mathbf{A}}$.

Consider a soft tissue strip that has been transformed from its undeformed reference configuration, Ω_0, to a deformed current configuration, Ω (Fig. 1a). Suppose, a point $\boldsymbol{x} = f(\boldsymbol{X})$ on the deformed body in Ω corresponds to the point $\boldsymbol{X}$ in Ω_0, then the deformation gradient ($\mathbf{F}$) is defined as:

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \tag{1}$$

Using $\mathbf{F}$, other important tensors in finite elasticity may be introduced such as the right Cauchy-Green deformation tensor, $\mathbf{C} = \mathbf{F}^T\mathbf{F}$; left Cauchy-Green deformation tensor, $\mathbf{B} = \mathbf{F}\mathbf{F}^T$; Green strain tensor, $\mathbf{E} = 1/2(\mathbf{C} - \mathbf{I})$; velocity gradient, $\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1}$; and rate of deformation tensor, $\mathbf{D} = 1/2(\mathbf{L}^T + \mathbf{L})$, where the $(\dot{-})$ represents the material time derivative of a tensor.

Some important stress tensors are: Cauchy stress tensor, $\mathbf{S}$; first Piola-Kirchhoff (PK) stress tensor, $\mathbf{P}$; and second PK stress tensor, $\mathbf{T}$. These tensors are related as: $\mathbf{S} = (1/\det(\mathbf{F}))\mathbf{F}\mathbf{T}\mathbf{F}^T, \mathbf{T} = \mathbf{P}\mathbf{F}^{-T}$. For a detailed study about tensors and finite elasticity reader may refer Holzapfel (2000).

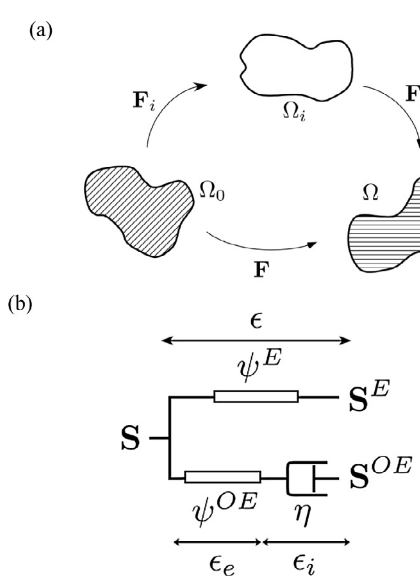

Fig. 1. (a) Motion of a continuum body in a viscoelastic framework and the decomposition of deformation gradient, $\mathbf{F}$, into an elastic, $\mathbf{F}_e$, and an inelastic, $\mathbf{F}_i$, part, respectively. (b) One-dimensional representation of the finite viscoelastic model. The total strain, ϵ, of the system is decomposed into elastic (ϵ_e) and inelastic (ϵ_i) parts linked with the in-series hyperelastic element and the dashpot, respectively.

2.1. Viscoelastic constitutive model

A thermodynamically consistent viscoelastic model suitable for finite deformation was proposed by Huber and Tsakmakis (2000). This model evolves from the concept of multiplicative decomposition of the deformation gradient and additive splitting of the total strain energy function (also known as the internal variable method (IVM), (Lubliner, 1985; Petiteau et al., 2013)). These decompositions can be defined as:

$$\mathbf{F} = \mathbf{F}_e\mathbf{F}_i \tag{2}$$

$$\psi = \psi^E + \psi^{OE} \tag{3}$$

where subscripts 'i' and 'e' indicate, respectively, the inelastic and elastic parts of a tensor.

Following the same analogy, the deformation, strain and velocity gradient tensors can also be decomposed into elastic and inelastic parts. Using Eq. (2), the relations $\mathbf{C}_e = \mathbf{F}_e^T\mathbf{F}_e, \mathbf{C}_i = \mathbf{F}_i^T\mathbf{F}_i, \mathbf{B}_e = \mathbf{F}_e\mathbf{F}_e^T, \mathbf{B}_i = \mathbf{F}_i\mathbf{F}_i^T, \mathbf{L}_e = \dot{\mathbf{F}}_e\mathbf{F}_e^{-1}$, and $\mathbf{L}_i = \dot{\mathbf{F}}_i\mathbf{F}_i^{-1}$ may be introduced. Some useful relations between these decomposed tensors are:

$$\mathbf{C} = \mathbf{F}_i^T\mathbf{C}_e\mathbf{F}_i \tag{4}$$

$$\mathbf{B}_e = \mathbf{F}\mathbf{C}_i^{-1}\mathbf{F}^T \tag{5}$$

$$\mathbf{L} = \mathbf{L}_e + \mathbf{F}_e\mathbf{L}_i\mathbf{F}_e^{-1} \tag{6}$$

In one-dimension (1D) this model may be thought of as a rheological model analogous to the SLS where both linear springs are replaced with hyperelastic elements as shown in Fig. 1b. The decomposed parts of total strain energy (see Eq. (3)) are associated with the parallel and in-series hyperelastic elements, respectively. The decomposition of $\mathbf{F}$ introduces an intermediate configuration (Ω_i) in between the reference and the current configuration, as shown in Fig. 1a, which would result when the stress is released at an infinitely fast rate from the current configuration of the body to a stress-free configuration. In Ω_i, the total strain (Γ), and its decomposed elastic (Γ_e) and inelastic (Γ_i) parts take the form:

$$\Gamma = \mathbf{F}_i^{-T}\mathbf{E}\mathbf{F}_i^{-1} \tag{7}$$

$$\Gamma_e = \frac{1}{2}(\mathbf{C}_e - \mathbf{I}), \quad \Gamma_i = \frac{1}{2}(\mathbf{I} - \mathbf{B}_i^{-1}) \tag{8}$$

where a tensor operation is carried out in Eq. (7) to transform $\mathbf{E}$ from Ω_0 to Ω_i. An interesting feature to be noted is that similar to the 1D SLS model, the total strain tensor in Ω_i decomposes additively into elastic and inelastic parts, i.e., $\Gamma = \Gamma_e + \Gamma_i$. Using Eqs. (2) and (7) the strain rate in Ω_i can be written as:

$$\overset{\triangle}{\Gamma} = \dot{\Gamma} + \mathbf{L}_i^T\Gamma + \Gamma\mathbf{L}_i = \mathbf{F}_i^{-T}\dot{\mathbf{E}}\mathbf{F}_i^{-1} = \mathbf{F}_e^T\mathbf{D}\mathbf{F}_e \tag{9}$$

where $\overset{\triangle}{\Gamma}$ is the covariant rate of strain tensor. Now, the rate of change of the strain energy functions can be derived as:

$$\frac{d}{dt}\left(\psi^E(\mathbf{E})\right) = \frac{\partial\psi^E}{\partial\mathbf{E}} : \dot{\mathbf{E}} = \frac{\partial\psi^E}{\partial\mathbf{E}} : \left(\mathbf{F}_i^T\overset{\triangle}{\Gamma}\mathbf{F}_i\right) = \left(\mathbf{F}_i\frac{\partial\psi^E}{\partial\mathbf{E}}\mathbf{F}_i^T\right) : \overset{\triangle}{\Gamma} \tag{10}$$

$$\frac{d}{dt}\left(\psi^{OE}(\Gamma_e)\right) = \frac{\partial\psi^{OE}}{\partial\Gamma_e} : \dot{\Gamma}_e = \frac{\partial\psi^{OE}}{\partial\Gamma_e} : \overset{\triangle}{\Gamma} - \mathbf{C}_e\frac{\partial\psi^{OE}}{\partial\Gamma_e} : \mathbf{L}_i \tag{11}$$

The second law of thermodynamics states that the total production of entropy per unit time is non-negative for all thermodynamic processes (Holzapfel, 2000). For a purely mechanical system in 3D, the second law of thermodynamics takes the form:

$$\mathbf{S} : \mathbf{D} - \dot{\psi} \geqslant 0 \tag{12}$$

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