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PARAMETER IDENTIFICATION TO ENFORCE PRACTICAL OBSERVABILITY OF NONLINEAR SYSTEMS

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Abstract: The sensitivity of measurements to unmeasured state variables strongly affects the rate of convergence of a state estimator. To overcome potential observability problems, the approach has been to identify the model parameters so as to reach a compromise between model accuracy and system observability. An objective function that weighs the relative importance of these two objectives has been proposed in the literature. However, this scheme relies on an extensive heuristic search to select the weighting coefficients. This paper proposes an objective function that is the product of measures of these two objectives, thus alleviating the need for the trial-and-error selection of the weighting coefficient. The proposed identification procedure is evaluated using both simulated and experimental data, and with different observer structures.

Keywords: Parameter identification, Observability, State estimation, Identification for estimation, Kalman filter, Particle filter

1. INTRODUCTION

Observability tests typically provide a binary yes/no answer and, thus, do not help assess whether practical observability problems such as slow convergence of the state estimates will occur. A study has shown that even an accurate bioprocess model can lead to poor state estimates when the measurements have a low sensitivity with respect to the unmeasured states (Bogaerts and Vande Wouwer, 2004). To alleviate this problem, the same authors have suggested a model "falsification" procedure, in which the model parameters are identified so as to achieve a compromise between model accuracy (via minimization of a criterion expressing the deviation between the model and plant states) and system observability (via a measure of observability based on sensitivity matrices). Unfortunately, the proposed objective function contains a weighting coefficient that is best determined via a trial-and-error procedure involving repeated optimization.

The contribution of this paper is to propose an objective function that (i) achieves the aforementioned compromise between model accuracy and system observability, and (ii) can be determined without trial-and-error procedure. It turns out that the objective function can be formulated as the product of two measures that are related to the sought objectives. This study also compares the classical extended Kalman filter (Maybeck, 1982) with a less classical (at least

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in bioprocess monitoring) particle filter (Doucet *et al.*, 2001) on two case studies, one in simulation and the other using experimental data.

The paper is organized as follows. Section 2 sets the notations for parameter identification and briefly reviews the concept of nonlinear system observability. Section 3 describes the parameter identification procedure for state estimation, while Section 4 details the results obtained with a simulated example and a real-life application. Finally, conclusions are provided in Section 5.

2. PRELIMINARIES

2.1 Parameter identification

We consider continuous-time nonlinear models associated with discrete-time measurements:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \ ) \qquad \mathbf{x}(0) = \mathbf{x}_0 \tag{1}$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}(t_k), \ ) \tag{2}$$

where $\mathbf{x}(t) \in \ ^{n_x}$ is the state vector, $\mathbf{u}(t) \in \ ^{n_u}$ the input vector and $\mathbf{y}_k \in \ ^{n_y}$ the output vector at the discrete time t_k. is the vector of parameters to be identified. $\mathbf{f}$ and $\mathbf{h}$ are, in general, nonlinear vector functions. For simplicity of notation, the time dependency of the signals $\mathbf{x}(t)$ and $\mathbf{u}(t)$ will be dropped in the sequel.

The parameter identification problem can be formulated as follows:

$$(\hat{\ }, \hat{\mathbf{x}}_0) = \arg \min_{,\mathbf{x}_0} J_{id}(\ , \mathbf{x}_0) \tag{3}$$

$$\text{given} \quad model \ (1)-(2); \ \mathbf{y}_{meas}$$

with

$$J_{id}(\ , \mathbf{x}_0) = \frac{1}{2\mathrm{N}} \ _{k=1}^{N} \left(\mathbf{y}_{meas,k} - \mathbf{y}_k(\ , \mathbf{x}_0)\right)^T Q_k^{-1} \left(\mathbf{y}_{meas,k} - \mathbf{y}_k(\ , \mathbf{x}_0)\right) \tag{4}$$

where $\mathbf{y}_{meas,k}$ represents the measured outputs at time t_k, Q_k the covariance matrix of the measurement noise, and N the data length. Note that, since the initial conditions are rarely known in practice, they can be considered as decision variables as well. This is similar to the approach taken in moving-horizon estimation (Haseltine and Rawlings, 2005).

The properties of the resulting model can be analyzed. In the context of the design of a state observer, system observability is of paramount importance.

2.2 Observability of nonlinear systems

A system is said to be completely observable if it is possible to reconstruct the state vector from a finite number of output measurements. Global observability analysis of nonlinear systems is a delicate task since observability generally depends on the system inputs. The analysis is made simpler through the introduction of canonical forms (Zeitz, 1984; Zeitz, 1989). A system is said to be globally observable if the nonlinear model can be expressed in the following canonical form (Gauthier and Kupka, 1994):

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{\mathbf{x}}^1 \\ \dots \\ \dot{\mathbf{x}}^i \\ \dots \\ \dot{\mathbf{x}}^{q-1} \\ \dot{\mathbf{x}}^q \end{bmatrix} = \begin{bmatrix} \mathbf{f}^1(\mathbf{x}^1, \mathbf{x}^2, \mathbf{u}) \\ \dots \\ \mathbf{f}^i(\mathbf{x}^1, \dots, \mathbf{x}^{i+1}, \mathbf{u}) \\ \dots \\ \mathbf{f}^{q-1}(\mathbf{x}^1, \dots, \mathbf{x}^q, \mathbf{u}) \\ \mathbf{f}^q(\mathbf{x}^1, \dots, \mathbf{x}^q, \mathbf{u}) \end{bmatrix}, \tag{5}$$

$$\mathbf{y} = \begin{bmatrix} h_1(x_1^1) \\ h_2(x_1^1, x_2^1) \\ \dots \\ h_{n_1}(x_1^1, \dots, x_{n_1}^1) \end{bmatrix} \tag{6}$$

$$\text{with} \quad \forall i \in \{1, \dots, q\} : \mathbf{x}^i = \left[x_1^i, \dots, x_{n_i}^i\right]^T,$$

$$n_1 \geq n_2 \geq \dots \geq n_q, \qquad \ _{1\leq i\leq q} n_i = n_x$$

and if the following conditions are satisfied:

$$\bullet \ \forall j \in \{1, \dots, n_1\} : \frac{\ h_j}{\ x_j^1} \neq 0 \tag{7}$$

$$\bullet \ \forall i \in \{1, \dots, q-1\}, \quad \forall (\mathbf{x}, \mathbf{u}) \in \ ^{n_x} \times \ ^{n_u} :$$

$$\text{rank}\left(\frac{\ \mathbf{f}^i(\mathbf{x}, \mathbf{u})}{\ \mathbf{x}^{i+1}}\right) = \mathrm{n_{i+1}} \tag{8}$$

This canonical form assumes that only the first state subvector $\mathbf{x}^1$ is measured, i.e. $n_y = n_1$. Condition (7) states that $\mathbf{x}^1$ can be inferred directly from the measurements, whereas condition (8) implies a pyramidal influence of the state subvector $\mathbf{x}^{i+1}$ on $\mathbf{x}^i$, so that any differences in the state trajectory can be detected in the measurements.

A convenient way to check condition (8) is to compute the $(n_{i+1}) \times (n_{i+1})$ matrix

$$M_i(\mathbf{x}, \mathbf{u}) = \left(\frac{\ \mathbf{f}^i(\mathbf{x}, \mathbf{u})}{\ \mathbf{x}^{i+1}}\right)^T \left(\frac{\ \mathbf{f}^i(\mathbf{x}, \mathbf{u})}{\ \mathbf{x}^{i+1}}\right) \tag{9}$$

and check the rank condition:

$$\text{rank}\left[M_i(\mathbf{x}, \mathbf{u})\right] = n_{i+1} \tag{10}$$

It is shown in (Bogaerts and Vande Wouwer, 2004) that an accurate process model can lead to poor estimates when the matrices $M_i(\mathbf{x}, \mathbf{u})$ are ill-conditioned, i.e. when the internal connections between state variables are somewhat "loose", at least in some time intervals. Upon analysis, this lack of connectivity is generally related to the model structure and the selection of operating conditions.

3. IDENTIFICATION FOR STATE ESTIMATION

This section discusses two ways of including the observability issue in the parameter identification: (i) a