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AN INTERVAL OBSERVER FOR NON-MONOTONE SYSTEMS: APPLICATION TO AN INDUSTRIAL ANAEROBIC DIGESTION PROCESS

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Abstract: This article proposes a new class of observers in order to estimate unmeasured variables of a biotechnological process. The observer is developed on the basis of interval estimates, which provide guaranteed upper and lower bounds of the unknown variables. The proposed method exploits monotonicity properties of the error dynamics. A bundle of observers is generated by appropriately varying the observer gains. The method is applied to a real industrial anaerobic digestion plant, for the estimation of the key variables on the basis of the available measurements of the methane flow rate.

Keywords: Interval observers, robust observers, anaerobic digestion, monotone systems.

1. INTRODUCTION

Biotechnological processes play an increasing role in many industries, such as food, pharmaceutical or depollution (Stephanopoulos *et al.*, 1998). In contrast with other kinds of processes that are perfectly described by physical laws -like mechanical or electrical systems- biotechnological processes are dealing with living organism. As a consequence, their modelling is uncertain and is known to have a lower aptitude to accurately match experimental results.

On the other hand, online monitoring of biotechnological processes is a very difficult task. The difficulty to measure chemical and biological variables is one of the main limitations in the improvement of monitoring and optimisation of bioreactors. The lack of hardware sensors to perform monitoring tasks has forced the implementation of complicated, and not reliable methods. This problem becomes of great importance in more complex systems like anaerobic wastewater treatment plants, where critical instability of the process must be avoided, making the monitoring of the system variables an important issue (Mailleret *et al.*, 2004).

As an efficient solution for the inherent problem of monitoring biotechnological processes, the internal state reconstruction can be achieved by formulating observers, also called software sensors.

Many types of observers have been proposed and extensively studied, even for nonlinear biological systems (Bastin and Dochain, 1990; Bernard and Gouzé, 2006) and the choice of the design method depends on the kind of available models. Indeed, the quality of the used model is a factor of great importance when choosing an observation strategy. For instance, if a well identified and validated model is available, a high gain observer (Gauthier *et al.*, 1992) may perform good estimations of the internal state. If we have to deal with large uncertainties in model parameters, inputs and measurements, robust state estimation methods, for example based on interval analysis (Jaulin *et al.*, 2001) and approximation of reachable set using ellipsoids (Kurzhanski and Valyi, 1997), both for discrete time systems, and cooperativity based interval observers (Gouzé *et al.*, 2000; Rapaport and Gouzé, 2003) for the continuous time systems, should provide robust estimates.

Interval observers work under the formulation of two observers: an upper observer, which produces an upper bound of the state vector, and a lower observer producing a lower bound, providing by this way a bounded interval in which the state vector is guaranteed to lie. For the formulation of the interval observer, it is necessary to know bounds of the uncertainties in the model (*i.e.* uncertainties in model parameters, input variables, etc.). These observers are based on hypotheses of monotonicity (Smith, 1995), and the design of such observers for non monotone systems is much less straightforward. We propose here an alternative design strategy to the one proposed in (Moisan and Bernard, 2005) for non monotone systems.

In this paper we propose a guaranteed interval estimation exploiting monotonicity properties of the error dynamics under two approaches. The first correspond to a direct cooperative observer and the second one to dynamics which becomes cooperative after a variable change. Both approaches require a specific choice for the signs of the observer gains. We run then several observers in parallel, obtaining a bundle of observers (Bernard and Gouzé, 2004), and we take the best estimate provided by the bundle.

This paper is organised as follows. In section 2 the considered class system is presented, linking it with example related to a general biotechnological model. Section 3 introduces the observer, considering first a perfect knowledge case and then a general uncertainty framework. Section 4 is devoted to running many observers in parallel to obtain the observer bundle. The application to an anaerobic wastewater treatment process is studied in Section 5.

2. CLASS OF SYSTEMS AND EXAMPLE

We consider a general class of nonlinear systems whose dynamics are expressed as follows:

$$(S): \begin{cases} \dot{\xi} = A\xi + Br(\xi) + d, \quad \xi(0) = \xi_0 \\ y = r(\xi) \end{cases} \qquad (1)$$

where $A \in \mathbb{R}^{n\times n}$ is a diagonal and stable matrix, $B \in \mathbb{R}^n$ and $d \in \mathbb{R}^n$ is a system input. For the sake of simplicity, in this paper we restrict the analysis to the nonlinear term $r(\xi)$ such that $r(\xi) \in \mathcal{C}^1 : \mathbb{R}^n \mapsto \mathbb{R}$.

An example of such a structure can be found in the classical mass balance models for continuously stirred bioprocesses as proposed by (Bastin and Dochain, 1990):

$$\dot{\xi} = -DH\xi + Kr(\xi) + D\xi_{in} - Q(\xi) \qquad (2)$$

In this model, the state vector $\xi = (\xi_1, \xi_2, \dots, \xi_n)^t$ is the vector of all the process concentrations and biomasses. The matrix K contains the stoichiometric coefficients, also known as yield coefficients of the model. The vector $r(\xi) = (r_1(\xi), r_2(\xi), \dots, r_k(\xi))^t$ is a vector of reaction rates (or conversion rates) representing the microbial activity (in this paper we only consider the case $r(\xi) \in \mathbb{R}$). The diagonal matrix H stands for the fraction of biomass or substrates in the liquid phase. The influent feeding concentration is represented by the positive vector ξ_{in}. The dilution rate $D > 0$ is the ratio of the influent flow rate and of the volume of the fermenter. Finally $Q(\xi)$ represents the gaseous exchange with the outside of the fermenter.

With such a classical modeling we have *e.g.*:

$$A = -DH, \quad B = K \text{ and } d = D\xi_{in} - Q(\xi)$$

The bacterial kinetics model ($r(\xi)$) is generally a rough approximation and is highly uncertain. Moreover, for wastewater treatment processes, the influent concentrations ξ_{in} are generally not measured.

The objective of this paper is to derive an interval observer for the class of systems (1) considering uncertainties in the measurements, in function $r(\xi)$, as well as uncertainties in the vector d.

Before introducing the observer, let us recall an useful definition.

Definition 1. A square matrix P is said to be cooperative if its offdiagonal terms are nonnegative (Smith, 1995): $p_{ij} \geq 0, \ \forall i \neq j$.

Remark 1. The operator $\leq$ applied between vectors or matrices should be understood as a set of inequalities applied component by component.

The main properties of a cooperative system defined by

$$\dot{X} = PX + b$$

where $X, b \in \mathbb{R}^n$, is that it keeps the (partial) order of the trajectories. If we consider two initial conditions $x^1(0)$ and $x^2(0)$ such that $x^1(0) \geq x^2(0)$, then $x^1(t, x^1(0)) \geq x^2(t, x^2(0)), \forall t \geq 0$.

An interval observer is proposed in (Rapaport and Gouzé, 2003) for system (1) provided that $r(\xi)$ is a monotone function. Here we want to extend these results in the case where $r(\xi)$ is not monotone. For this purpose we will use the following property.

Property 1. The Lipschitz function r can be rewritten as the difference of f and g which are two increasing functions of ξ:

$$r(\xi) = f(\xi) - g(\xi)$$