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ADVANCED DYNAMICAL RISK ANALYSIS FOR MONITORING ANAEROBIC DIGESTION PROCESS

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Abstract: In this paper we study an unstable biological process used for waste water treatment. This ecosystem can have two locally stable equilibria and an unstable one. We first introduce the model and recall its main properties. We then study the phase plane and split it into fifteen zones according to the sign of the derivatives of the state vector. Then we propose a methodology to monitor in real-time the trajectory of the system across these zones and determine its position in the plane. We finally define a dynamical risk index based on the transitions from one zone to another and classify the zones according to their dangerousness. It is worth noting that the proposed approach do not rely on the value of the parameters and is thus very robust.

Keywords: Anaerobic digestion, non-linear system diagnosis, Haldane model, risk index.

1. INTRODUCTION AND MOTIVATION

Anaerobic digestion is a more and more popular bioprocess (Angelidaki et al. [2003]) that at the same time treats wastewater and produces energy through methane (CH_4) and hydrogen. This complex ecosystem involves more than 140 bacterial species (Delbès et al. [2001]) that progressively degrade the organic matter into carbon dioxide (CO_2) and methane (CH_4). However this process is known to be very delicate to manage since it is unstable (Fripiat et al. [1984]): an accumulation of intermediate compounds can lead to the acidification of the digester.

The objective of this paper is to analyze and characterize the dynamics of such an unstable system, in order to better assess the risk of going toward process acidification. In a previous paper (Hess and Bernard [Submitted]), a static criterion based on the size of the attraction basin of the useful working point was proposed. This criterion was however independent of the real state of the process. Here we propose to identify, on the basis of the qualitative analysis of some monitored signals (like e.g. methane flow rate) the region of the state space where the system lies. The robustness of these regions to an increase of the loading is then studied and leads to a dynamical risk criterion.
The proposed methodology relies on a mathematical analysis, but as in Hess et al. [2006] it is devoted to be applied to on-line monitoring of industrial processes.

The paper is composed as follows: in the second section the dynamical model considered for anaerobic digestion process is detailed. The third part puts the emphasis on the analysis of the model

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dynamics. Transitions graphs that describe the possible commutations between regions are produced in a fourth part. Finally, a risk criterion to assess the stability of the process is proposed.

2. MODEL DEFINITION AND ANALYSIS

2.1 Model presentation

We consider a generic macroscopic model of a one-stage anaerobic process where an organic substrate s is degraded by a bacterial consortium x into methane:

$$k\, s \xrightarrow{\mu(.)\, x} x + k_G\, CH_4$$

The dynamical mass-balance model in a continuous fixed-bed reactor is straightforwardly derived:

$$\begin{cases} \dot{s} &= D\,(s_{in} - s) - k\,\mu(.)\,x \\ \dot{x} &= \mu(.)\,x - \alpha\,D\,x \\ q_m &= k_G\,\mu(.)\,x \end{cases} \tag{1}$$

where D is the dilution rate, s_{in} the influent substrate concentration, α the fraction of free biomass in the liquid medium (the rest of the biomass is attached to a support), q_m the methane molar flow rate, and, k and k_G are pseudo-stoichiometric coefficients representing respectively the substrate degradation and the methane production yields. $\mu(.)$ is the bacterial growth rate.

We will consider an Haldane-like growth rate:

$$\mu(s) = \mu_m \frac{s}{s + k_s + \frac{s^2}{k_i}} \tag{2}$$

k_i being the inhibition constant, k_s the half-saturation one and μ_m the specific growth rate.

2.2 Positivity and boundedness

This type of model has been widely described for $\alpha = 1$(Andrews [1968]) and we only recall the main properties of system (1).

For the rest of our study we assume that D and s_{in} are piecewise constant. Hence D, s_{in} and the initial values of the variables are positive.

Property 1. System (1) with initial conditions $\xi_0 = (s_0, x_0)$ in $\mathbb{R}^2_+$ remains positive and bounded at all time.

Proof: We only give the sketch of the proof.
Since $\mu(0) = 0$, it is trivial to show the positivity of the system. To show the boundedness we have to consider the variable $z = s + k\,x$ and study its dynamics as exposed in Hess et al. [2006].

In the next section we present a short analysis of the model equilibria and its stability.

2.3 Equilibrium and stability

Considering the non-monotonic growth rate of equation (2), System (1) is close to a generic Haldane model, but slightly modified by the term α. Equilibria $\xi^* = (s^*, x^*)$ other than the acidification $\xi^\dagger = (s_{in}, 0)$ are solution of the following system:

$$\begin{cases} \mu(s^*) &= \alpha D \\ x^* &= \frac{s_{in} - s^*}{\alpha k} \\ \mu(s) &= \mu_m \frac{s}{s + k_s + \frac{s^2}{k_i}} \end{cases} \tag{3}$$

Several cases are possible (Hess et al. [2006]), depending on the parameters and operating conditions; one case of special interest is when there are two steady-states in the interior domain [2], one of which along with the acidification one is locally stable (*c.f.* Figure 1).

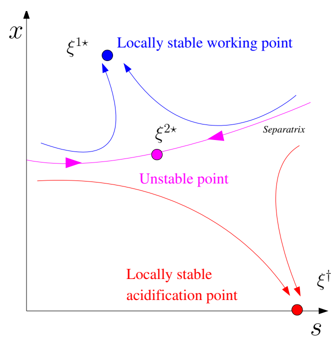

Fig. 1. Possible orbits in the phase plane.

Definition For $\xi^{1\star}$, the interior critical point of system (1), we define its **basin of attraction** $\mathcal{B}^{1\star}$as the set of initial conditions in $\mathbb{R}^2_+$ converging asymptotically towards it:

$$\mathcal{B}^{1\star} = \left\{ \xi_0 \in \mathbb{R}^2_+ \mid \lim_{t\to+\infty} \xi(\xi_0, t) = \xi^{1\star} \right\},$$

We define in the same way for $\xi^\dagger$:

$$\mathcal{B}^\dagger = \left\{ \xi_0 \in \mathbb{R}^2_+ \mid \lim_{t\to+\infty} \xi(\xi_0, t) = \xi^\dagger \right\}$$

The separatrix is defined as the variety dividing the plane into the two attraction basins (it is indeed the stable manifold associated to the unstable steady state ξ^{2*}).

Ideally the plant manager should choose operating conditions that maximise $\mathcal{B}^{1\star}$, the attraction basin of the normal operating mode $\xi^{1\star}$, in order to limit the risk of destabilisation. Yet in practice the operator has to simultaneously process a maximal loading and prevent a plant failure.

Being able to determine on what side of the separatrix the system evolves is capital to ensure

[2] here we do not consider the simpler case where there is only one interior equilibrium which is globally stable.