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How do people reduce compound lotteries?

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ABSTRACT

This paper provides experimental evidence for a rather important question: How do people reduce compound lotteries? As an alternative to the reduction of compound lotteries axiom of expected utility, I also test the compound independence axiom that can be employed by several decision theories. While the non-parametric test does not reject the compound independence axiom, I do not find support for evaluation of compound lotteries by the compound independence axiom through rank dependent utility that was used to motivate the axiom. The reduction of compound lotteries axiom is tested by two methods used in the literature. The validity of the axiom depends on the particular method used. While binary choices support the validity of the reduction axiom, there is no evidence of evaluation of compound lotteries through the axiom. Furthermore, out-of-sample predictions indicate that expected value is the best predictor of elicited certainty equivalents of compound lotteries. Interestingly, expected utility is the best predictor of elicited certainty equivalents for simple lotteries. The results suggest that subjects follow different mechanisms when evaluating compound lotteries as compared to simple ones.

Introduction

A compound lottery refers to a lottery that allows the outcomes to be lotteries themselves. Representing risky situations by compound lotteries is closer to real life situations, especially if we consider the opportunity cost of our choices. The reduction of compound lotteries axiom of expected utility (EU) states that individuals are indifferent between a compound lottery and it's actuarially equivalent simple lottery that generate the same probability distribution over outcomes. To state this formally, let $S_1, S_2, S_3, \ldots$ denote simple lotteries in a set φ. The preference relation denoted by $\succeq$ is a binary relation on the φ which allows the comparison of pairs.

Axiom 1. Reduction of compound lotteries (ROCL): Let $S_1 = (x_1, p_1; \ldots; x_i, p_i; \ldots; x_n, p_n)$ and $S_2 = (y_1, q_1; \ldots; y_i, q_i; \ldots; y_n, q_n)$. Then, for all $S_1, S_2 \in \varphi$ and $r \in (0, 1)$ we have:

$$C = (S_1, r; S_2, 1 - r) \sim (x_1, rp_1; \ldots; x_n, rp_n; y_1, q_1(1 - r); \ldots; y_n, q_n(1 - r)).$$

Thus, any compound lottery is equivalent to its reduced form lottery which has been called the actuarially equivalent lottery. Empirical evidence has not always supported the ROCL axiom. See, for instance, Bar-Hillel (1973), Kahneman and Tversky (1979), Bernasconi and Loomes (1992), Miao and Zhong (2012), Abdellaoui et al. (2015), and Bernasconi and Bernhofer (2017). This evidence has been produced mainly by two methods. The first method uses binary choices to test the consistency of choices when a compound lottery is replaced by its actuarially equivalent lottery (Harrison et al., 2015). The second method is based on comparing the elicited certainty equivalents of compound lotteries and their actuarially equivalent ones (Abdellaoui et al., 2015; Miao and Zhong, 2012).

Since there are many studies (Hershey and Schoemaker, 1985; Johnson and Schkade, 1989; Lichtenstein and Slovic, 2006), illustrating that subjects are not consistent through different question frames, it is reasonable to test the ROCL axiom by both methods and check for the robustness of the result. Moreover, in this way we can examine the phenomena of preference reversal (Lichtenstein and Slovic, 1971) in a choice between a compound lottery and its actuarially equivalent lottery.

Alternatively, from a theoretical point of view, Segal (1990) introduced the compound independence axiom that does not require the reduction principle. The compound independence implies that decision makers evaluate compound lotteries using the certainty equivalents of its possible first-stage outcomes (Bernasconi, 1994).[1] The compound independence axiom is the independence axiom adapted to two-stage lotteries.

Axiom 2. Compound independence (CI) Consider the two-stage compound lottery $C_1 = (S_1, r; S_3, 1 - r)$ and $C_2 = (S_2, r; S_3, 1 - r)$. The preference relation $\succeq$ satisfies the compound independence axiom if for all $S_1, S_2, S_3 \in \varphi$ and $r \in (0, 1)$ we have:

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[1] Note that the certainty equivalents can be generated by any decision theory, hence, making the compound independence axiom compatible with most preference relations.

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Table 1
Lottery combinations.

pairs	C Lotteries; $C = (S, r; x, 1-r)$							AE Lotteries				S Lotteries				EV
	Probabilities		$S = (y_1, q; y_2, 1-q)$				x	Outcomes		Probabilities		Outcomes		Probabilities		
	r	$1-r$	y_1	y_2	q	$1-q$		Low	High	Low	High	Low	High	Low	High	
1	0.3	0.7	0	30	0.5	0.5	0	0	30	0.85	0.15	0	15	0.7	0.3	4.5
2	0.3	0.7	5	25	0.5	0.5	5	5	25	0.85	0.15	5	15	0.7	0.3	8
3	0.3	0.7	10	20	0.5	0.5	10	10	20	0.85	0.15	10	15	0.7	0.3	11.5
4	0.3	0.7	5	10	0.5	0.5	5	5	10	0.85	0.15	5	7.5	0.7	0.3	5.75
5	0.3	0.7	5	20	0.5	0.5	5	5	20	0.85	0.15	5	12.5	0.7	0.3	7.25
6	0.7	0.3	0	30	0.5	0.5	0	0	30	0.65	0.35	0	15	0.3	0.7	10.5
7	0.7	0.3	5	25	0.5	0.5	5	5	25	0.65	0.35	5	15	0.3	0.7	12
8	0.7	0.3	10	20	0.5	0.5	10	10	20	0.65	0.35	10	15	0.3	0.7	13.5
9	0.7	0.3	5	10	0.5	0.5	5	5	10	0.65	0.35	5	7.5	0.3	0.7	6.75
10	0.7	0.3	5	20	0.5	0.5	5	5	20	0.65	0.35	5	12.5	0.3	0.7	10.25

$C_1 \succeq C_2$ if and only if $S_1 \succeq S_2$.

The CI axiom has been used as a justification for the validity of random-lottery incentive mechanism in which the ROCL axiom of EU is violated (Starmer and Sugden, 1991). If individuals satisfy the CI axiom, then, the *isolation effect* is at work. The isolation effect was introduced by Kahneman and Tversky (1979) as a simplifying process that involves disregarding the common components of alternatives. Tests of the isolation effect seem to be sensitive to the representation of risky situations. When risky lotteries are transparent and the independence axiom is brought to the surface, the evidence supports the isolation effect (Tversky and Kahneman, 1981, 1986; Conlisk, 1989; Bernasconi, 1994; Cubitt et al., 1998). However, in more complex settings (Harrison et al., 2015) or when the comparison is between simple and two-stage/multi-stage lotteries, the isolation effect is violated (Cox et al., 2014, 2015; Hopfensitz and Van Winden, 2008).

When both axioms are tested directly, there seems to be stronger evidence supporting the CI axiom as compared to the ROCL (Kahneman and Tversky, 1979; Bernasconi, 1992, 1994). For example, Bernasconi (1994) finds that while 57% of choices are consistent with the CI axiom, 43% of them are consistent with the ROCL axiom. When considering different forms of independence axiom, Bernasconi (1992) finds less impressive violations of the CI axiom as opposed to the ROCL or the mixture independence that requires the reduction principal.

The methodology to test the CI axiom is constructed based on the methods used to test the ROCL axiom. Hence, I test if the preference ordering of two compound lotteries such as C_1 over C_2 follows the same preference ordering as the distinguishing simple lotteries S_1 over S_2. Second, using out-of-sample predictions, I test if there is a significant difference between elicited certainty equivalents of compound lotteries and those predicted by the CI axiom through rank dependent utility (RDU) of Quiggin (1982). This is because Segal (1990, p. 375) used this method to explain the pattern of preferences in problem 3 and 4 of Kahneman and Tversky (1979) that are based on the *Allais paradox*.

The results of this experiment fails to reject the CI axiom. However, the data does not support the reduction of compound lotteries by the substitution of certainty equivalents through RDU. The results show that the validity of the reduction axiom is sensitive to the method used to test the axiom. While binary choices are consistent with the predictions of the ROCL axiom, I find differences in the evaluation of compound and actuarially equivalent lotteries, thus rejecting the ROCL axiom with the second method of analysis. This is consistent with the general finding of more violation in the valuation task than in the choice task (see, for example, Schmidt and Trautmann (2014) and Harbaugh et al. (2010)).

Furthermore, while the expected value appears to be the best predictor of the certainty equivalent of compound lotteries, expected utility has the most predictive power for simple lotteries. These results suggest that subjects evaluate simple and compound lotteries differently.

1. Experimental design

The experiment had two stages. Stage I was designed for testing the ROCL and the CI axioms by collecting subjects' choices between lotteries. The evaluation mechanism of compound lotteries was tested in stage II. In particular, stage II consists of eliciting certainty equivalents of a number of simple and two-stage compound lotteries.

In stage I, subjects were asked to make a series of choices between lottery pairs. However, they had the indifferent option as well. Following the methodology of Harrison et al. (2015) for testing the ROCL axiom, three types of lotteries were needed. These are compound lotteries (C), their associated actuarially equivalent lotteries (AE) and simple lotteries (S) that are used to construct three pairs of lotteries. Namely, a $AE-C$ pair, a $S-C$ pair and a $S-AE$ pair. I repeat this process 10 times. Hence, there are 10 pairwise choices in each pair. The lotteries used to construct 30 pairwise choices are presented in Table 1.

The ROCL axiom requires decision makers to be indifferent between a given C lottery and its associated AE lottery. Hence, when confronted with a choice from the $S-C$ pair, the pattern of preferences should be the same as the choice from the $S-AE$ pair. That is if a subject prefers C to S, s/he should prefer AE to S.

In order to test the CI axiom, I collect the pattern of preferences in a set of choices between pairs of compound lotteries C_1 and C_2 which have two distinguishing simple lotteries S_1 and S_2 in the second stage. I also collect the pattern of preferences between S_1 and S_2. Hence, I define $C_1 = (S_1, r; x, 1-r)$ and $C_2 = (S_2, r; x, 1-r)$ where S_1 and S_2 are the distinguishing simple lotteries. The CI axiom implies that the preference ordering of the two compound lotteries C_1 and C_2 follows the same preference ordering as the distinguishing simple lotteries S_1 and S_2. Table 2 shows these lottery pairs. While the $S_1 - S_2$ pairs have 10 choices, 5 of them are equivalent to S-AE choices. Hence, there were 45 choice questions.

In stage II, certainty equivalents of 30 simple and 10 compound lotteries were elicited. The 10 aforementioned C and AE lotteries used in stage I are included in this stage. This is to test subjects' consistency through different question frames and the evaluation mechanism of compound lotteries. The remaining 20 simple lotteries were designed such that the expected value of simple lotteries would follow a normal distribution while covering a wide range of probabilities and outcomes (see Table 3).

Each lottery in stage II was described and displayed by a decision tree with a subsequent list of 20 equally spaced guaranteed amounts. For each guaranteed amount, subjects had to indicate whether they prefer the guaranteed amount or the lottery (see Appendix A). The lottery's certainty equivalent was calculated as the mean of the guaranteed amount that was offered when the subject switched preferences