ARTICLE IN PRESS
JID: JEBO [m3Gsc;February 1, 2018;22:54]

Journal of Economic Behavior and Organization 000 (2018) 1–19

Contents lists available at ScienceDirect

Journal of Economic Behavior and Organization

journal homepage: www.elsevier.com/locate/jebo

A contest success function for networks[☆]

Irem Bozbay[a,*], Alberto Vesperoni[b]

[a] University of Surrey, School of Economics, Guildford, GU2 7XH, UK
[b] University of Klagenfurt, Institute of Economics, Klagenfurt, 9020, Austria

ARTICLE INFO

Article history:
Received 16 August 2016
Revised 12 January 2018
Accepted 23 January 2018
Available online xxx

JEL classification:
C70
D72
D74
D85

Keywords:
Conflict
Contest
Success function
Network
Pairwise stability

ABSTRACT

This paper models conflict as a contest within a network of friendships and enmities. We assume that each player is either in a friendly or in an antagonistic relation with every other player and players compete for winning by exerting costly efforts. We axiomatically characterize a success function which determines the win probability of each player given the efforts and the network of relations. In an extension, we allow for varying intensities of friendships and enmities. This framework allows for the study of strategic incentives and friendship formation under conflict as well as the application of stability concepts of network theory to contests.

1. Introduction

In many situations of conflict, we often observe that competing parties join forces to fight together against others or refrain from fighting with each other. For instance, lobby groups may cooperate in supporting the same legislation when their interests coincide; political parties may refrain from campaigning against each other when they have a common opportunity; belligerent states may form alliances for joint action if they face a common threat and so on. These parties do not necessarily act in a perfectly coordinated way, especially when their relation is an occasional opportunistic cooperation rather than a long term commitment. Such relations usually rely on informal bilateral agreements and may lead to a complex network.

This paper models conflict as a contest, where players compete for increasing their win probabilities by exerting costly efforts. While doing so, each pair of players may be in a *friendly* relation and abstain from competing against each other (their default state being antagonism), and the friendship relations between all pairs define a network. In this setting, we propose and axiomatically characterize a *success function* which determines the probability of winning for each player given all efforts and the network of relations. So far, the axiomatic work in the contest literature has exclusively focused on con-

☆ We are grateful to Çağatay Kayı, Kai Konrad, Dan Kovenock, Jingfeng Lu, Johannes Münster, Stergios Skaperdas and Karl Wärneryd for their valuable comments. Bozbay thanks Faculty of Arts and Social Sciences of the University of Surrey for the travel grant.

* Corresponding author.
E-mail address: i.bozbay@surrey.ac.uk (I. Bozbay).

https://doi.org/10.1016/j.jebo.2018.01.022
0167-2681/

Please cite this article as: I. Bozbay, A. Vesperoni, A contest success function for networks, Journal of Economic Behavior and Organization (2018), https://doi.org/10.1016/j.jebo.2018.01.022

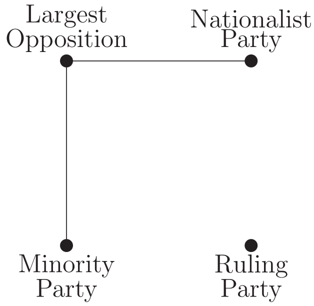

Fig. 1. An example network for political competition. We consider an electoral competition with 4 parties: the ruling party, the largest opposition, the nationalist party and the party representing a minority. A link between two parties represents friendship, and the political competition in this country leads to a network where a friend of a friend is an enemy.

flict between groups (e.g., Münster, 2009; Cubel and Sanchez-Pages, 2016) or between individuals (e.g., Skaperdas, 1996). In the former, players are divided into mutually exclusive groups (coalitions) and groups compete with each other, while in the latter each player competes individually against all others. Both approaches are crucial for the study of a broad set of environments. Yet, many competitive situations may lead to networks different than the *all against all* or *groups against groups* types of networks. For instance, in international relations most alliances between states do not mean perfect coordination or long term commitments. The opportunistic nature of tactical alliances[1] does not exclude a partnership between states which are not members of the same coalition, and the friend of a friend can be an enemy at times. Political competition may also lead to a complex network of relations. Despite the zero-sum nature of political gains, we see that a political party does not necessarily target all its opponents during campaigning and it might have a preference for which opponent to damage. In Fig. 1, we illustrate a network representing a four party political competition. In this example, the largest opposition party does not campaign against the nationalist party and the minority party hoping that there may be a vote switch from the centrist ruling party to them, which increases the largest opposition's chance of winning the election. However, a coalition of opposition parties fails to emerge given the historical conflict between the nationalist party and the minority party. This 'missing link' should intuitively make the ruling party better off compared to facing a coalition of opponents, while also strengthening the relative power of the largest opposition party within the opposition (being the 'hub' of the opposition). We aim to capture such strategic features of competition in our model. Our paper extends the axiomatic foundations of success functions to contests with any type of network of relations. As a starting point, we propose a class of success functions which we derive through a probabilistic argument following well-known results by McFadden (1973). We simply show that a player's probability of winning is equivalent to the probability that the *effective strength* of the player, which is an additively separable function of the efforts of her friends and enemies, is higher than anyone else's effective strength perturbed by random noise. This class is very general and particularly convenient in a variety of applications. To understand how win probabilities depend on the relations as well as efforts, we provide an axiomatic characterization. Our characterization consists of six axioms, three of which are direct extensions of the well-known anonymity, monotonicity of efforts and exhaustivity axioms in the literature. We define three new axioms; namely, the *monotonicity of relations*, the *independence of efforts of commons* (IEC), and the *independence of relations of others* (IRO), all of which incorporate the effect of the network variable on the success function. The monotonicity of relations simply imposes that befriending a player with higher effort than one's own leads to an increase in the probability of winning. We define our two independence axioms on the *relative* win probability of two players; in other words, on the ratio of their win probabilities. IEC states that the relative win probability of two players is independent of the efforts of their common friends and common enemies. In the context of the example in Fig. 1, IEC imposes that the relative win probabilities of the minority party and the nationalist party will not change by increased campaigning efforts of the largest opposition party (although each probability will increase). Hence, the effort of the largest opposition affects both parties similarly, so that their relative win probability remains constant. Our final axiom, IRO, allows for making across network comparisons and together with monotonicity of relations, it identifies how probabilities change in response to a changing relation. It implies that the *rate of change* of a relative probability as a result of a new friendship or enmity (the ratio of the new relative probability to the old) remains the same across all pairs of networks which differ only by the new relation. Once we restrict our attention to all against all contests, our functional form belongs to the well-established class characterized by Skaperdas (1996).[2] For contests between groups, our class does not immediately link to the class axiomatized in Münster (2009) as our function determines win probabilities of individual players rather than groups. However, the function obtained by summing up the win probabilities of group members derived by our function belongs to the class axiomatized in Münster (2009).

[1] In his categorization of strategic alliances, Ghez (2011) defines the tactical alliance as a form of state alignment which occurs when states encounter a common immediate threat. For further examples of tactical alliances and definitions of other categories, see Ghez (2011).

[2] The success functions characterized in Skaperdas (1996) had been widely applied to represent conflict. For seminal contributions, see, e.g., Haavelmo (1954), Tullock (1975) and Rosen (1986).