AN AFFINE GEOMETRICAL APPROACH TO POWER SYSTEMS PROBLEMS

Emmanuel D. Crainic[a] **, Alexander I. Petroianu**[b*]

[a] *Logiciels de réseaux, Institut de recherche d'Hydro-Québec, IREQ, 1800 Blvd. Lionel Boulet, Varennes, QC, Canada, J3X 1S1*
[b] *Department of Electrical Engineering, University of Cape Town, University Private Bag, Rondebosch 7701, Republic of South Africa*

Abstract: The paper introduces modern concepts and tools from affine geometry into power system analysis. It is shown that such an approach allows: i) a new non linear formulation of such classical problems as load flow and state estimation, ii) a more efficient way of solving such problems through non iterative methods. The new approach is illustrated for a small but representative example of a load flow for a two-bus network.

Keywords: affine geometry, affine transformations, linear methods, algebraic and geometric invariants, power systems load flow.

1. AFFINE GEOMETRY: AN INTRODUCTION

In many power system applications, some of the physical quantities involved in the formulation and the solution of the related problems are expressed either as complex numbers of a 1-dimensional complex space $\boldsymbol{C}^1$ or as vectors of a 2-dimensional real space $\boldsymbol{R}^2$. Both approaches have their proper merit. In the complex space $\boldsymbol{C}^1$ all the four arithmetic operations among the set of its complex numbers i.e.: addition, subtraction, multiplication and division are allowed. The elements of a real space $\boldsymbol{R}^2$, called vectors, are regarded as entities involved in linear operations, i.e.: multiplication of a vector by a scalar and the addition of two vectors. This vector space is a linear space. However, this may constitute a limitation, since a vector space contains only vectors of the same nature (for instance, power, or voltage or current vectors, etc., but not a combination of two or more vectors of different nature). In this paper, the authors emphasize the geometric aspects and the physical meaning of the affine space associated with the vector space usually used in the investigation of power system load flow problem (Petroianu, 1969).

A 2-dimensional affine space E is a space of points associated to a vector space $\vec{E}$ of the same dimension, in the sense that: **i)** for each pair of points *(a, b)* $\in E$, the difference *(a-b)* between them is a vector $\overrightarrow{AB}$ in the vector space $\vec{E}$, **ii)** For each vector in the vector space $\vec{E}$ and for each point in the affine space E, adding the vector to this point results in an another point in the affine space E, **iii)** every triplet of points *(a, b, c)* $\in E$ satisfies the relationship *(a-b)* + *(b-c)* = *(a-b)*. Therefore, there is an one-to-one mapping of the elements of the two associated spaces (Beklémichev, 1988).

An affine space E may be visualized itself as a linear space by choosing in it an arbitrary point O, called the origin, and in the appropriate vector space $\vec{E}$ a basis ($\mathbf{e}_1$, $\mathbf{e}_2$). If a is an arbitrary point in E, together with the coordinate origin O, it defines a vector $\overrightarrow{OA} \in \vec{E}$, the radius vector of the point a, which in terms of the basis ($\mathbf{e}_1$, $\mathbf{e}_2$), may be expressed as $\overrightarrow{OA} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2$. The coefficients x_1, x_2 are called the affine coordinates of the point a. As any space, the affine space is defined by its geometry. In the spirit of the *Erlangen Programm,* insisting on the concept of the *group* rather than that of the space, Klein (1974) saw any geometry, including the affine geometry, as the study of invariants under a group of transformations. An *affine transformation*, as linear mapping from an

* Corresponding author: Alexander I. Petroianu,
E-mail address: apetroianu@ebe.uct.ac.za

affine space to another affine space (or to itself), is any transformation that preserves parallelism of lines and the ratio of distances between colinear points. In a 2-dimensional affine space, to map a point (x, y) to a point (x', y') four main affine transformations, or linear combinations of them, may be used (Klein, 2004):

1. – rotation by an angle φ counter clockwise about the origin,
$$x' = x\cos\varphi + y\sin\varphi \quad (1)$$
$$y' = -x\sin\varphi + y\cos\varphi \quad (2)$$
2. – reflection in the x axis,
$$x' = x \quad (3)$$
$$y' = -y \quad (4)$$
3. – scaling,
$$x' = \lambda_x x \quad (5)$$
$$y' = \lambda_y y \quad (6)$$
4. – translation,
$$x' = x + \mu_x \quad (7)$$
$$y' = y + \mu_y \quad (8)$$

The affine transformations make the general affine group GA (2, ***R***), which is a semidirect product of the general linear group and the translations in E by vectors of $\vec{E}$. The essential difference between an affine and a vector space consists in the fact that in the affine space the operation of adding a vector to a point is allowed. The operations solely on points are also possible, but only under certain conditions: this is the subject of barycentric calculus (see Mõbius (1827) or Delode (2000)). An affine space, not being dependent on a specific choice of a coordinate system, is the appropriate framework in dealing with motions, trajectories, and physical or electromagnetical forces, among other things.

2. AFFINE GEOMETRICAL APPROACH TO THE LOAD FLOW PROBLEM

2.1 Power system load flow formulation

In power system analysis, the class of problems related to load flow (planning and operating versions) is of a mathematically non-linear type. In planning environment, the load flow problem assumes the knowledge of power injections and values of electrical parameters of network elements. The solution consists in finding the nodal voltages (module and angle). For a component of the network, for example a line (Figure 1), the apparent power flows are expressed as follows:

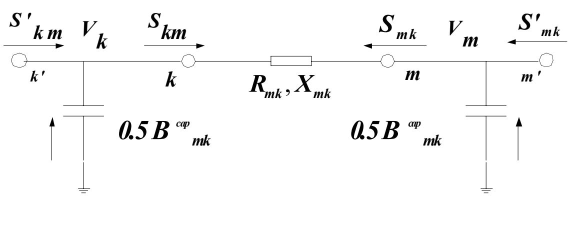

Fig. 1. Electrical line representation: π model

$$\underline{S}'_{mk} = \underline{S}_{mk} - i0.5V^2_m B^{cap}_{mk} \quad (9)$$
$$\underline{S}'_{km} = \underline{S}_{km} - i0.5V^2_k B^{cap}_{mk} \quad (10)$$

In the equations (9) and (10) the voltages are complex numbers represented by their modules and angles, i.e. V_m, V_k respectively, δ_m, δ_k with the difference:

$$\theta_{mk} = \delta_m - \delta_k \quad (11)$$

considered to be positive (all over this paper the hypothesis is made that m is the sending and k the receiving nodes of the active power). Analytically, the complex powers $\underline{S}_{mk}$ and $\underline{S}_{km}$ are as follows:

$$\underline{S}_{mk} = \underline{V}_m(\underline{V}^*_m - \underline{V}^*_k)(g_{mk} + ib_{mk}) = P_{mk} + iQ_{mk} \quad (12)$$
$$\underline{S}_{km} = \underline{V}_k(\underline{V}^*_k - \underline{V}^*_m)(g_{mk} + ib_{mk}) = P_{km} + iQ_{km} \quad (13)$$

where

$$g_{mk} = R_{mk}/(R^2_{mk} + X^2_{mk}) \quad (14)$$
$$b_{mk} = X_{mk}/(R^2_{mk} + X^2_{mk}) \quad (15)$$

In (12) and (13) P_{mk}, P_{km} are the active and Q_{mk}, Q_{km} the reactive powers. They represent the real and the imaginary parts of the complex numbers $\underline{S}_{mk}$ and $\underline{S}_{km}$:

$$P_{mk} = -V_mV_k\cos\theta_{mk}g_{mk} + V_mV_k\sin\theta_{mk}b_{mk} + V^2_m g_{mk} \quad (16)$$
$$Q_{mk} = -V_mV_k\sin\theta_{mk}g_{mk} - V_mV_k\cos\theta_{mk}b_{mk} + V^2_m b_{mk} \quad (17)$$

$$P_{km} = -V_mV_k\cos\theta_{mk}g_{mk} - V_mV_k\sin\theta_{mk}b_{mk} + V^2_k g_{mk} \quad (18)$$
$$Q_{km} = V_mV_k\sin\theta_{mk}g_{mk} - V_mV_k\cos\theta_{mk}b_{mk} + V^2_k b_{mk} \quad (19)$$

From the above expressions, the active and reactive powers in (9) and (10) are:

$$P'_{mk} = P_{mk} \quad (20)$$
$$P'_{km} = P_{km} \quad (21)$$
$$Q'_{mk} = Q_{mk} - 0.5V^2_m B^{cap}_{mk} \quad (22)$$
$$Q'_{km} = Q_{km} - 0.5V^2_k B^{cap}_{mk} \quad (23)$$

With a known voltage (module and angle) at a chosen reference bus, the system to be solved has 2(N-1) non linear equations of bus *power injections*, expressed as sums of adjacent power transit of type (16) to (19), and (N-1) *voltage modules* and (N-1) *voltage angles* as variables. Its iterative solution is a well known procedure (for a detailed treatment of it, see Debs (1988) or Eremia, *et al*, (2000)).

2.2 Bus voltage module

By taking into account (14) and (15), the angle γ_{mk} is defined as:

$$\tan\gamma_{mk} = b_{mk} / g_{mk} \quad (24)$$

With the radius ρ_{mk} of the circle (see Figure 2) expressed as:

$$\rho_{mk} = \sqrt{g^2_{mk} + b^2_{mk}} \quad (25)$$

the following trigonometric functions may be derived for the angle $2\gamma_{mk}$:

$$\cos 2\gamma_{mk} = (g^2_{mk} - b^2_{mk}) / \rho^2_{mk} \quad (26)$$
$$\sin 2\gamma_{mk} = (2g_{mk}b_{mk}) / \rho^2_{mk} \quad (27)$$