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Volatility clustering: A nonlinear theoretical approach

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ABSTRACT

This paper verifies the endogenous mechanism and economic intuition on volatility clustering using the coexistence of two locally stable attractors proposed by Gaunersdorfer et al. (2008). By considering a simple asset pricing model with two types of boundedly rational traders, fundamentalists and trend followers, and noise traders, we provide theoretical conditions on the coexistence of a locally stable steady state and a locally stable invariant circle of the underlying nonlinear deterministic financial market model and show numerically that the interaction of the coexistence of the deterministic dynamics and noise processes can endogenously generate volatility clustering and long range dependence in volatility observed in financial markets. Economically, volatility clustering occurs when neither the fundamental nor trend following traders dominate the market and when traders switch more often between the two strategies.

1. Introduction

Volatility clustering, one of the most important stylized facts in financial markets, refers to the observation that large changes in price tend to be followed by large changes and small changes tend to be followed by small changes. In other words, asset price fluctuations display irregular interchanging between high volatility and low volatility episodes. Since it was first observed by Mandelbrot (1963) in commodity prices, volatility clustering has been widely observed and documented in stocks, market indices and exchange rates. Despite the extensive development of various statistical models following the ARCH and GARCH models pioneered by Engle (1982) and Bollerslev (1986), these models offer very limited economical explanation of the mechanism in generating the volatility clustering.

Recent development of asset pricing models based on boundedly rational traders with heterogeneous beliefs has proposed a number of mechanism explanations. In particular, Gaunersdorfer et al. (2008) propose an endogenous mechanism and economic intuition based on the coexistence of a stable steady state and a stable limit circle. In this paper, we consider a simple asset pricing model with two types of boundedly rational traders, fundamentalists and trend followers, and noise traders. By applying the normal form analysis and the centre manifold theory on the underlying nonlinear deterministic model, we provide theoretical conditions on the coexistence of a stable steady state and a stable closed invariant circle. When buffeted with noises, the stochastic model can endogenously generate volatility clustering and long range dependence in volatility observed in financial markets. Economically, with strong trading activities of either the fundamental investors

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or the trend followers, market price fluctuates around either the fundamental value with low volatility or a cyclical price movement with high volatility depending on market conditions. With the fundamental noise and noise traders, this triggers an irregular switching between two volatility regimes and therefore leads to volatility clustering. In particular, the effect becomes more significant when traders switch their strategies more often. We therefore verify the endogenous mechanism on volatility clustering proposed by Gaunersdorfer et al. (2008) and provide an economic explanation on the volatility clustering.

Gaunersdorfer et al. (2008) propose two generic mechanisms to explain the volatility clustering: one is the coexistence of a stable steady state and a stable limit circle and the other is the intermittency and associated bifurcation routes to strange attractors. They provide very nice economic intuitions on how the mechanisms could be used to explain the volatility clustering and why these phenomena arise in nonlinear evolutionary financial systems. The main idea behind the first proposed mechanism of Gaunersdorfer et al. (2008) is that a locally stable steady state and a locally stable closed invariant circle coexist in a nonlinear financial market system. When the price is attracted by the stable steady state, that is when the fundamentalists are more active in the market, the price is stable and the corresponding return is less volatile. However when the trend followers exhibit strong trading activity, the market price is attracted by the stable circle, leading to large price fluctuations and high volatility in returns. Buffeted with noises, the price process switches irregularly between the two stable regions from time to time. As a result, the irregular switchings between the two attractors triggered by large random shocks lead the return process to exhibit volatility clustering. Mathematically, Gaunersdorfer et al. (2008) demonstrate the coexistence through a Chenciner bifurcation, a codimension-two bifurcation in which two parameters vary simultaneously. Near the Chenciner bifurcation point, there exists an open region, called "volatility clustering region", in a two-dimensional parameter subspace in which a stable steady state and a stable limit circle coexist. Due to the complexity of the normal form analysis of codimension-two bifurcation, they identify the volatility clustering region numerically and indicate the potential of the mechanism in generating volatility clustering. The proposed mechanism has also been used to explain path dependent coordination of expectations in asset pricing experiments. In the learning-to-forecast laboratory experiments, Hommes et al. (2005) find three different types of aggregate asset price behaviour: monotonic convergence to the stable fundamental steady state, dampened price oscillations, and permanent price oscillations. Motivated by the mechanism of Gaunersdorfer et al. (2008), Agliari et al. (2016) develop a simple behaviour model with switching and explain individual as well as the three different types of aggregate behaviour in the experiments through the coexistence mechanism. Moreover, Chenciner bifurcation is also observed in other economic models[1] and thus can be regarded as an important feature of nonlinear economic models.

In this paper, we obtain analytical conditions on the coexistence of a stable steady state and a stable closed invariant circle and provide a systematic way to identify the "volatility clustering interval" by examining the stability of a Neimark–Sacker bifurcation through the change of one parameter.[2] More explicitly, we first apply the stability and bifurcation analysis to examine the local stability of the steady state with respect to a Neimark–Sacker bifurcation parameter. We then investigate the direction and the stability of the bifurcated closed invariant circle by applying the normal form method and the centre manifold theory. The coexistence is then jointly determined by the conditions when the steady state is locally stable and the bifurcated circle is backward and unstable. In this case, the bifurcated unstable circle can be extended backward with respect to the bifurcation parameter until a threshold value and then the extended circle becomes forward and stable. Therefore, the stable steady state coexists with the stable 'forward extended' circle, in between the 'backward extended' circle is unstable. Correspondingly, there exists an interval for the bifurcation parameter in which the two locally stable attractors coexist. This implies that, even when the fundamental steady state is locally stable, prices need not converge to the fundamental value, but may settle down to a stable limit circle, depending on the initial conditions. Based on the conditions on the coexistence, we further demonstrate numerically that the stochastic model is able to generate various stylized facts, including non-normality in asset returns, volatility clustering, and long range dependence in volatility observed in financial markets. Therefore we provide theoretical foundation and numerically supporting evidence on the proposed mechanism of Gaunersdorfer et al. (2008).

This paper contributes to the heterogeneous agent models (HAMs) literature by providing a better understanding of the global dynamics of the underlying nonlinear deterministic financial market model, the interaction between deterministic global dynamics and noises, and therefore complexity of financial market behaviour. Following the seminal work of Brock and Hommes (1997, 1998), various HAMs have been developed to incorporate adaptation, evolution, heterogeneity, and even learning with both Walrasian and market maker market clearing scenarios.[3] Those models have successfully explained various market behaviour (such as market booms and crashes, long deviations of the market price from the fundamental price), the stylized facts (such as skewness, kurtosis, volatility clustering and fat tails of returns), and power laws behaviour,

[1] See, for example, Neugart and Tuinstra (2003), Agliari (2006) and Lines and Westerhoff (2010).

[2] Gaunersdorfer et al. (2008) illustrate the coexistence through a Chenciner bifurcation, which requires Hopf bifurcation together with the condition on the first Lyapunov coefficient $a(0)=0$. The advantage of our method over the analysis merely based on the Chenciner bifurcation is that we can detect systematically the coexistence interval. The advantage is that the analysis of both the direction and stability of the bifurcation is based only on one parameter. In general, both the bifurcation direction and stability are determined by different conditions. However for our model, they are completely determined by the first Lyapunov coefficient ($a(0)$), which significantly simplifies our analysis.

[3] See, for example, the market maker scenario in Farmer and Joshi (2002) and Chiarella and He (2003); the impact of heterogeneous risk aversion and learning in Chiarella and He (2002); the dynamics of moving averages in Chiarella et al. (2006); and complex price dynamics within a multi-asset market framework in Westerhoff (2004).