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On the learning and stability of mixed strategy Nash equilibria in games of strategic substitutes

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ABSTRACT

This paper analyzes the learning and stability of mixed strategy Nash equilibria in games of strategic substitutes (GSS), complementing recent work done in the case of strategic complements (GSC). Mixed strategies in GSS are of particular interest because it is well known that such games need not exhibit pure strategy Nash equilibria. First, we establish bounds on the strategy space which indicate where randomizing behavior may occur in equilibrium. Second, we show that mixed strategy Nash equilibria are generally unstable under a wide variety of learning rules. Multiple examples are given.

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1. Introduction

Many positive results have been established in the literature on games of strategic substitutes (GSS) in terms of the characterization of solution sets, adaptive learning processes, and comparative statics properties.[1] The analysis of this wide class of games, however, has concentrated mainly on situations where players are assumed to play pure strategies only, and although it is well known that such games need not exhibit pure strategy Nash equilibria (PSNE), the role of mixed strategies has largely been ignored. It is therefore important to explore conditions under which players may find it optimal to randomize over their set of actions, and if mixed strategy Nash equilibria (MSNE) offer good long-run predictions of behavior. By drawing on a connection in GSS between learning in repeated play and rationalizability, it is first shown that under very general conditions, players may learn to play in a manner consistent with mixed behavior. As a consequence, a sufficient condition for global stability is obtained. The main result, however, confirms that MSNE with non-degenerate support do not generally offer good predictions by showing that they are unstable under a wide range of learning rules.

The validity of MSNE as an equilibrium prediction has long been a topic of discussion in economics. The classical argument against them is as follows: If opponents are behaving in such a way as to make a player indifferent between a subset of her actions, why would randomizing be preferred to simply choosing a pure strategy best response? One response to this argument has been by way of Harsayni's Purification Theorem, which proves that if players privately observe a sequence

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[1] See Dubey et al. (2006), Roy and Sabarwal (2010), and Roy and Sabarwal (2012), for example.

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of i.i.d. random shocks to their payoffs, then a mixed equilibrium emerges in the resulting game of incomplete information which approximates the original mixed equilibrium.[2]

More recent studies have asked whether, if randomizing behavior is to be understood in the framework of players committing to a distribution over their actions when an underlying game is repeated over time, players can eventually learn to play according to an equilibrium distribution. Work along these lines has been conducted in a variety of game-theoretic settings. Crawford (1985) shows that purely mixed strategy Nash equilibria are always unstable under gradient dynamics.[3] Fudenberg and Kreps (1993), Kaniovski and Young (1995), and Benaim and Hirsch (1999) study the convergence to mixed equilibria in 2×2 games and 3×2 games, whereas Ellison and Fudenberg (2000) study the stability of MSNE in 3×3 games. Hofbauer and Hopkins (2005) investigates such stability in 2-player, finite-action games under a smooth fictitious play learning process, and Benaim et al. (2009) study convergence in games whose Nash equilibria are mixed and unstable under fictitious play-like learning. The notion of stable MSNE has also found important applications in large population games, specifically in the context of price dispersion. Hopkins (2008) describes, and gives evidence for, the phenomenon of price dispersion, where different sellers of a homogenous product charge different prices. These situations often arise due to incomplete information among consumers as to who the lowest price seller is, and can be described by mixed strategy dispersed price equilibria. Hopkins and Seymour (2002) find mixed results as to the stability of such equilibria, showing that when consumer behavior is fixed, convergence is possible in some cases. On the other hand, Lahkar (2011) finds that all dispersed price equilibria are unstable under perturbed best response dynamics, while Lahkar and Riedel (2014) find that they are not generally stable under logit dynamics.

This paper is most closely related to Echenique and Edlin (2004), which considers the stability of mixed strategy Nash equilibria in games of strategic complements (GSC) when the set of players is finite and action spaces are a complete lattice. The heart of the analysis lies in exploiting a complementarity between the order structure inherent in GSC and a quite general assumption on how players update their beliefs, which includes Cournot and fictitious play learning. Specifically, if a player makes a small mistake in her beliefs about equilibrium behavior by shifting an arbitrarily small amount of probability towards the largest action in the support of opponents' MSNE profile, then this upward shift (in FOSD) of beliefs implies that she will best respond by playing a strategy higher than her equilibrium mixed strategy. A subsequent update in beliefs again results in an even higher upward shift in FOSD, resulting in an even higher response. This pattern continues on indefinitely, so that intended play never returns to the original MSNE. A similar argument can be made when the underlying game is a GSS, as the next example illustrates.

Example 1. Consider the following slight variation to the 3-player Dove–Hawk–Chicken game presented in Roy and Sabarwal (2010):

P3 = D:

P1 \ P2	D	H
D	$1, 1, 1$	$\varepsilon, 2, 1$
H	$2, 1, \varepsilon$	$1, \varepsilon, 1$

P3 = H:

P1 \ P2	D	H
D	$1, \varepsilon, 2$	$1, 1, \varepsilon$
H	$\varepsilon, 1, 1$	$0, 0, 0$

where $\varepsilon \in (0, 1)$. This is a GSS which has no PSNE. One would hope, therefore, that a MSNE would provide a good prediction of play. After calculating the best-response functions, we obtain:

$$BR_1 = \begin{cases} D & \sigma_3(D) < \left(\frac{1-\varepsilon\sigma_2(D)}{2-\varepsilon}\right) \\ [0,1] & \sigma_3(D) = \left(\frac{1-\varepsilon\sigma_2(D)}{2-\varepsilon}\right) \\ H & \sigma_3(D) > \left(\frac{1-\varepsilon\sigma_2(D)}{2-\varepsilon}\right) \end{cases}, \quad BR_2 = \begin{cases} D & \sigma_1(D) < \left(\frac{1-\varepsilon\sigma_3(D)}{2-\varepsilon}\right) \\ [0,1] & \sigma_1(D) = \left(\frac{1-\varepsilon\sigma_3(D)}{2-\varepsilon}\right) \\ H & \sigma_1(D) > \left(\frac{1-\varepsilon\sigma_3(D)}{2-\varepsilon}\right) \end{cases}, \quad BR_3 = \begin{cases} D & \sigma_2(D) < \left(\frac{1-\varepsilon\sigma_1(D)}{2-\varepsilon}\right) \\ [0,1] & \sigma_2(D) = \left(\frac{1-\varepsilon\sigma_1(D)}{2-\varepsilon}\right) \\ H & \sigma_2(D) > \left(\frac{1-\varepsilon\sigma_1(D)}{2-\varepsilon}\right) \end{cases}$$

We see that when player i believes that her opponents are playing $\sigma_{-i} = \left(\left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{1}{2}\right)\right)$, then $\sigma_i = \left(\frac{1}{2}, \frac{1}{2}\right)$ is a best response. That is, σ defined by $\forall i \in I, \sigma_i = \left(\frac{1}{2}, \frac{1}{2}\right)$ is a MSNE. Now suppose that players make a slight error in their judgement about the behavior of others, so that for $\alpha > 0$ small, player i believes that all other players $j \neq i$ will play $\sigma_j = \left(\frac{1}{2} - \alpha, \frac{1}{2} + \alpha\right)$. Then in the first round, each player best-responds uniquely by playing D, or $\sigma_i = (1, 0)$. If players are Cournot learners, so that in each successive round they best-respond only to the profile played in the previous round, then in the second round, each player i best responds uniquely by playing H, or $\sigma_i = (0, 1)$. Continuing in this manner, we see that play therefore enters a cycle: (D, D, D), (H, H, H), (D, D, D), . . ., etc., and it never again becomes optimal to best respond by mixing evenly among

[2] See Govindan et al. (2003) for a shorter and more general proof of this result.
[3] As opposed to the best-response dynamics studied here. See Jordan (1993) for a discussion.

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