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Reduction of compound lotteries with objective probabilities: Theory and evidence☆

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ABSTRACT

The reduction of compound lotteries axiom (ROCL) has assumed a central role in the evaluation of behavior toward risk and uncertainty. We present experimental evidence on its validity in the domain of objective probabilities. Our battery of lottery pairs includes simple one-stage lotteries, two-stages compound lotteries, and their actuarially equivalent one-stage lotteries. We find *violations of ROCL* and that behavior is better characterized by a source-dependent version of the Rank-Dependent Utility model rather than Expected Utility Theory. Since we use the popular "1-in-K" random lottery incentive mechanism payment procedure in our main test, our experiment explicitly recognizes the impact that this payment procedure may have on preferences. Thus we also collect data using the "1-in-1" payment procedure. We *do not infer any violations of ROCL* when subjects are only given one decision to make. These results are supported by both structural estimation of latent preferences as well as non-parametric analysis of choice patterns. The random lottery incentive mechanism, used as payment protocol, itself induces an additional layer of "compounding" by design that might create confounds in tests of ROCL. Therefore, we provide a word of caution for experimenters interested in studying ROCL for other purposes, such as the relationship between ambiguity attitudes and attitudes toward compound lotteries, to carefully think about the design to study ROCL, payment protocols and their interaction with the preferences being elicited.

The reduction of compound lotteries axiom (ROCL) has assumed a central role in the evaluation of behavior toward risk, uncertainty and ambiguity. We present experimental evidence on its validity in domains defined over objective probabilities, where the tests are as clean as possible.[1] Even in this setting, one has to pay close attention to the experimental payment protocols used and their interaction with the experimental task, so that one does not inadvertently introduce confounds

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[1] The validity of ROCL over objective probabilities has also been identified as a potential indicator of attitudes toward uncertainty and ambiguity. Smith (1969) conjectured that people might have similar, source-dependent preferences over compound lotteries defined over objective probabilities *and* over ambiguous lotteries where the probabilities are not well-defined. Halevy (2007) provides experimental evidence that attitudes toward ambiguity and compound objective lotteries are indeed tightly associated. Abdellaoui et al. (2015) find that the latter relationship is weaker in their experiment.

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that may contaminate hypothesis testing. Using the popular random lottery incentive mechanism (RLIM) we find violations of ROCL, but when RLIM is not used we find that behavior is consistent with ROCL.

We therefore show that a fundamental methodological problem with tests of the ROCL assumption is that one cannot use an incentive structure that may induce subjects to behave in a way that could be confounded with violations of ROCL. This means, in effect, that experimental tests of ROCL must be conducted with each subject making only one choice.[2] Apart from the expense and time of collecting data at such a pace, this also means that evaluations must be on a between-subjects basis, in turn implying the necessity of modeling assumptions about heterogeneity in behavior.

In Sections 1 and 2 we define the theory and experimental tasks used to examine ROCL in the context of objective probabilities. In Section 3 we present evidence from our experiment. We find *violations of ROCL*, and observed behavior is better characterized by the Rank-Dependent Utility model (RDU) rather than Expected Utility Theory (EUT). However, *violations of ROCL* only occur when *many* choices are given to each subject and RLIM is used as the payment protocol. We do *not infer any violations of ROCL* when subjects are each given only *one* decision to make. Section 4 draws conclusions for modeling, experimental design, and inference about decision making.

1. Theory

We start with a statement of some basic axioms used in models of decision-making under risk, and then discuss their implications for the experimental design. Our primary conclusion is the existence of an interaction of usual experimental payment protocols and the validity of ROCL. To understand how one can design theoretically clean tests of ROCL that do not run into confounds, we must state the axioms precisely.

1.1. Basic axioms

Following Segal (1988, 1990, 1992), we distinguish between three axioms: the reduction of compound lotteries axiom (ROCL), the compound independence axiom (CIA) and the mixture independence axiom (MIA).

The ROCL states that a decision-maker is indifferent between a two-stage compound lottery and the actuarially equivalent simple lottery in which the probabilities of the two stages of the compound lottery have been multiplied out. With notation to be used to state all axioms, let X, Y and Z denote simple lotteries, A and B denote two-stage compound lotteries, express strict preference, and express indifference. Then the ROCL axiom says that $A \sim X$ if the probabilities and prizes in X are the actuarially equivalent probabilities and prizes from A. Thus if A is the compound lottery that pays in a first stage $100 if a coin flip is a head and $50 if the coin flip is a tail and in a second stage pays “double or nothing” of each possible outcome of the first stage with a 50:50 chance, then X would be the lottery that pays $200 with probability ½ × ½ = ¼, $100 with probability ½ × ½ = ¼, and nothing with probability ½.[3] To use the language of Samuelson (1952; p. 671), a compound lottery generates a *compound income–probability–situation*, and its corresponding actuarially equivalent single-stage lottery defines an *associated income–probability–situation*, and that “. . .only algebra, not human behavior, is involved in this definition.” From an observational perspective, one must then see choices between compound lotteries and actuarially equivalent simple lotteries to test ROCL.

The CIA states that two compound lotteries, each formed from a simple lottery by adding a positive common lottery with the same probability, will exhibit the same preference ordering as the simple lotteries. In other words, the CIA states that if A is the compound lottery giving the simple lottery X with probability α and the simple lottery Z with probability $(1-\alpha)$, and B is the compound lottery giving the simple lottery Y with probability α and the simple lottery Z with probability $(1-\alpha)$, then $A > B$ iff $X > Y$, $\forall \alpha \in (0,1)$. It says nothing about how the compound lotteries are to be evaluated, and in particular *it does not assume ROCL*: it only restricts the preference *ordering* of the two constructed compound lotteries to match the preference *ordering* of the original simple lotteries.[4]

Finally, the MIA says that the preference ordering of two simple lotteries must be the same as the actuarially equivalent simple lottery formed by adding a common outcome in a compound lottery of each of the simple lotteries, where the common outcome has the same value and the same (compound lottery) probability. More formally, the MIA says that $X > Y$ iff the actuarially equivalent simple lottery of $\alpha X + (1-\alpha)Z$ is strictly preferred to the actuarially equivalent simple lottery of $\alpha Y + (1-\alpha)Z$, $\forall \alpha \in (0,1)$. Stated so, it is clear the MIA strengthens the CIA by making a definite statement that the constructed

[2] One alternative is to present the decision maker with several tasks at once and evaluate the portfolio chosen, or to present the decision maker with several tasks in sequence and account for wealth effects. Neither is attractive, since they each raise a number of (fascinating) theoretical confounds to the interpretation of observed behavior. One uninteresting alternative is not to pay the decision maker for the outcomes of the task.

[3] Formally, compound lottery A pays either $100 or $50 with equal chance in the first stage; in the second “double or nothing” stage it pays $200 or nothing with equal chance if the outcome of the first stage is $100, and pays $100 or nothing with equal chance if the outcome of the first stage is $50. This compound lottery reduces to a single-stage lottery X that pays $200, $100 or $0 with 25%, 25% and 50%, respectively.

[4] Segal (1992; p.170) defines the CIA by assuming that the second-stage lotteries are replaced by their certainty-equivalent, “throwing away” information about the second-stage probabilities before one examines the first-stage probabilities at all. Hence one cannot then *define* the actuarially equivalent simple lottery, by construction, since the informational bridge to that calculation has been burnt. The certainty-equivalent could have been generated by any model of decision making under risk, such as RDU or Prospect Theory.