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# On the fragility of sunspot equilibria under learning and evolutionary dynamics☆

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A B S T R A C T

In this article, we investigate the possibility of sunspot equilibria to emerge from a process of learning and adaptation on agents' beliefs. We consider both finite state Markov sunspots and sunspots in autoregressive form, and derive conditions for the existence of an heterogeneous equilibrium where only a fraction of agents condition their forecasts on the sunspot. We then show that when the fraction of agents conditioning their forecasts on a sunspot changes over time, under evolutionary dynamics, these restrictions need to evolve endogenously with population shares. We argue that such requirement questions the possibility of sunspot equilibria to emerge through a process of evolution and adaptation on agents' beliefs: in order for a sunspot equilibrium to emerge, all agents must simultaneously coordinate on the same sunspot variable at the same time.



## 1. Introduction

Sunspot equilibria are an intriguing possibility, as they open the door to fluctuations in economic activity driven purely by agents' expectations and disconnected from economic fundamentals. From the seminal works of Azariadis (1981) and Cass and Shell (1983), the possibility of self-fulfilling equilibria is well known among economists: because agents expect some particular state of the system to get realized in the future, that very state emerges as an equilibrium outcome for the economy.

While the early works considered the possibility of finite state Markov sunspot equilibria (and in particular 2 state sunspot equilibria – 2-SSE), in the business cycles literature a different class of sunspot solutions is more frequently considered, with an autoregressive-moving average (ARMA) form. Examples are found in McCallum (1983) and Farmer (1993).

As Evans and McGough (2011) recently remarked, the fact that sunspot equilibria are theoretically possible in a model does not make them necessarily relevant from an economic perspective, as it might not be possible for agents to coordinate on such equilibria. Because of this, a number of authors have tried to understand the conditions under which sunspot equilibria are learnable. Woodford (1990), Evans and Honkapohja (1994a) and Evans and Honkapohja (2003a, 2003b) analyze learnability

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for finite state Markov sunspot equilibria, while Evans and Honkapohja (1994b) and Evans and McGough (2005a,b) show that also sunspot solutions in ARMA form can be learnable. In particular, Evans and McGough (2005a) demonstrate how in this last case the representation of the solution is crucial for its learnability properties. Building on this result, Evans and McGough (2011) show in a purely forward looking model that when finite state Markov sunspots equilibria are stable under learning, all sunspot equilibria are, provided a common factor representation is used.

All these works take a representative agent approach, and consider only the possibility of all agents conditioning their expectations on an extraneous sunspot component. Heterogeneity in expectations, though, has attracted increasing interest in the recent literature, as it is recognized that it represents a real world feature that economists must take into account in their understanding of expectations formation. In particular, the possibility of different predictors being endogenously chosen on the basis of their relative performance has been investigated in different contexts. From the seminal work of Brock and Hommes (1997), a number of works have analyzed the evolutionary selection of forecasting rules and their impact on economic outcomes. Recent examples include Branch and Evans (2006), Hommes (2009), Guse (2010) and Berardi (2011).

Much less investigated so far has been the link between heterogeneity and sunspot equilibria. A notable exception is Berardi (2009), who shows the possibility of heterogeneous equilibria, where only a fraction of agents use a sunspot variable in their forecasts, to emerge in a purely forward looking model, but who also points out the fragility of such equilibria under predictor choice dynamics. If agents are allowed to choose endogenously whether to include or not a sunspot in their forecasting model, based on a mean squared error measure of performance, it does not exist an equilibrium where only a fraction of agents uses the sunspot.

The aim of this article is to study the conditions that are required for a sunspot equilibrium to emerge from a process of learning and adaptation on agents' beliefs. The main contribution will be to provide general results about the fragility of sunspot equilibria when learning and evolutionary dynamics are taken into account. In particular, we will show that evolutionary dynamics are intrinsically incompatible with sunspot equilibria, as they require the (exogenous) sunspot's statistical properties to endogenously change together with population dynamics.

Friedman (1991) advocates for evolutionary dynamics as models of repeated anonymous strategic interaction, where actions that are more "fit", given the distribution of behaviors, tend over time to displace actions that lead to lower rewards. In our setting, the "game" is between agents conditioning their forecasts only on fundamentals, and agents using also a sunspot variable in their model. Would either group prevail in the long run? Note that those agents using only fundamentals have a model that is in fact mispecified (or underparameterized), since the sunspot, through the expectations of the other group, enters into the dynamics of the economy. It will turn out that the answer to our question will not depend on the stability of the evolutionary process but will be instead of a more general nature: evolutionary dynamics kill the possibility of sunspot equilibria altogether, by imposing a time-varying restriction between population dynamics and sunspots.

We will present our argument both for finite state Markov sunspots and for sunspots in autoregressive form and show that the results are not a feature of the particular form assumed for the sunspot, but depend instead on the restrictions that evolutionary dynamics impose on it.

We will model agents' endogenous selection of forecasting rules by using replicator dynamics, which represents the evolution of the fraction of agents using each of the possible predictors available. The concept of replicator dynamics is popular in game theory and it is used to model evolutionary dynamics of strategies in the population of players. While it is borrowed from biology, where it was first introduced by Taylor and Jonker (1978) to formalize the notion of evolutionarily stable strategy, Borgers and Sarin (1997) give it a learning interpretation at the individual level. Fudenberg and Levine (1998) provide an extensive treatment in game theory, while Sethi and Franke (1995), Branch and McGough (2008) and Guse (2010) have applied it to macroeconomic settings. We will then show in a later section that our results do not hinge on the specific choice of model for evolutionary dynamics, and present our argument using the Brock and Hommes (1997) model of evolution in predictor choices.

In order to pin down parameters in each forecasting model, or perceived law of motion (PLM), we will follow a growing literature in macroeconomics and assume that agents act as econometrician and recurrently estimate those parameters using techniques such as recursive least-squares (for a detailed treatment of the concepts and techniques used in this literature, see Evans and Honkapohja, 2001). In equilibrium, parameter values in agents' forecasting models will therefore minimize the mean squared error for that specific class of models.

The relationship between evolutionary and adaptive learning dynamics in this work can therefore be thought of as one of slow-fast adjustment: adaptive learning takes place at a faster speed than evolutionary dynamics, so that parameters in each model are at their least squares values (the value that minimizes the mean squared errors for each model) when evolutionary dynamics take place. Intuitively, agents tend to fine-tune often the model they use (update of parameter values through adaptive learning), but only infrequently abandon altogether the model they are using and switch to another model (evolutionary dynamics on model selection).

The plan of the paper is as follows: in Section 2 we present finite state Markov sunspot equilibria and extend them to a heterogeneous setting; in Section 3 we present equilibria with sunspots in AR(1) form and extend them to a heterogeneous setting; in Section 4 we introduce evolutionary dynamics, show how heterogeneity impacts on the resonant condition for the existence of sunspot equilibria and derive implications for the possibility of sunspot equilibria to emerge endogenously in an economy; Section 5 considers alternative predictor choice dynamics; Section 6 concludes.