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Original

Stability analysis of a laser with two modulated saturable absorbers

Oscar Andres Naranjo-Montoya

Centro de Investigaciones en Óptica (CIO), Loma del Bosque 115, Lomas del Campestre, León, Guanajuato, CP 37150, Mexico

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Abstract

The stability analysis of a model for a laser system with two modulated saturable absorbers, each one modulated at a different frequency, is performed. The model is based on four equations describing the temporal evolution of the photon flux, the population inversion in the active media, and the saturation coefficients of each saturable absorber. The system dynamics is discussed in order to find stable system control regions.

Keywords: Stability analysis; Laser system; Modulated saturable absorbers

1. Introduction

The dynamics of a laser system with two saturable absorbers (see Fig. 1) can be described by a model based on the Statz–DeMars equations, which originally were developed to describe oscillations in a Maser (Statz & DeMars, 1960). This model has undergone many modifications to be adopted for laser systems (Tarassov, 1985; Tang & Statz, 1963; Thompson & Malacara, 2001). For a complete phenomenological description of a laser with two saturable absorbers, only four equations are needed: the photon-flux equation, an equation for the population inversion density in the active medium and two saturable population inversion equations that give the saturation coefficient for each saturable absorber (Wilson & Aboites, 2013). Therefore, the Statz–DeMars equations for a three level laser system with two saturable absorbers without modulation are written as follows:

$$\begin{aligned}
\frac{dS}{dt} &= \Gamma\nu\sigma NS - \Gamma\nu\frac{L_{\alpha_1}}{L_m}k_{\alpha_1}S - \Gamma\nu\frac{L_{\alpha_2}}{L_m}k_{\alpha_2}S - \frac{1}{T}S\\
\frac{dN}{dt} &= -\beta\frac{\sigma}{\hbar\omega}NS + \frac{N_0 - N}{\tau}\\
\frac{dk_{\alpha_1}}{dt} &= -\frac{2\sigma_{\alpha_1}k_{\alpha_1}S}{\hbar\omega} + \frac{k_{0\alpha_1} - k_{\alpha_1}}{\tau_{\alpha_1}}\\
\frac{dk_{\alpha_2}}{dt} &= -\frac{2\sigma_{\alpha_2}k_{\alpha_2}S}{\hbar\omega} + \frac{k_{0\alpha_2} - k_{\alpha_2}}{\tau_{\alpha_2}},
\end{aligned} \tag{1}$$

where S is the emitted photon density, N is the population inversion of the active medium, k_{α_1} and k_{α_2} are the resonant absorptions of the saturable absorbers 1 and 2 respectively, σ_{α_1} and σ_{α_2} are the saturable absorbers cross-sections, and N_{α_1} and N_{α_2} are the population inversions of the saturable absorbers ($k_{\alpha_1} = -\sigma_{\alpha_1}N_{\alpha_1}$ and $k_{\alpha_2} = -\sigma_{\alpha_2}N_{\alpha_2}$). Γ, ν, σ and T stand, respectively, for cavity filling coefficient, optical frequency, active medium cross-section and photon lifetime in the cavity; β is the coefficient which accounts for the difference in population inversion coursed by lasing; L_m, L_{α_1} and L_{α_2} are, respectively, the active medium and the saturable absorbers lengths; $k_{0\alpha_1}$ and $k_{0\alpha_2}$ are the linear resonant saturable absorbers absorption coefficients without lasing; N_0 is

E-mail address: oscarnaranjo@cio.mx

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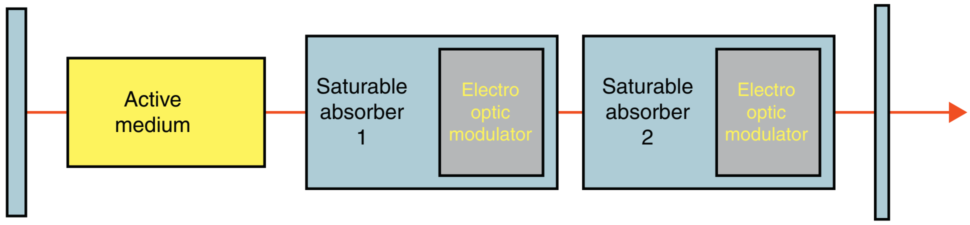

Fig. 1. Three-level laser system with two saturable absorbers with electro-optic modulators.

the population inversion in the active medium without radiation; τ, τ_{α_1} and τ_{α_2} stand for relaxation time in the active medium and in the saturable absorbers, respectively; finally, $\hbar\omega$ is the photon energy (Aboites & Ramírez, 1989).

Assuming that the two saturable absorbers have equal relaxation times ($\tau_{\alpha_1} = \tau_{\alpha_2} = \tau_\alpha$), and defining the next adimensional parameters and variables: $t' = t/\tau, G = \tau/t, \delta = \tau/\tau_\alpha, \rho_1 = 2\sigma_{\alpha_1}/\beta\sigma, \rho_2 = 2\sigma_{\alpha_2}/\beta\sigma, \alpha = \Gamma\nu\sigma TN$ and $\alpha_{\alpha_1} = -\Gamma\nu Tk_{0\alpha_1}L_{\alpha_1}/L_m = -\Gamma\nu T\sigma_{\alpha_1}n_{\alpha_1}/L_m,\ \alpha_{\alpha_2} = -\Gamma\nu Tk_{0\alpha_2}L_{\alpha_2}/L_m = -\Gamma\nu T\sigma_{\alpha_2}n_{\alpha_2}/L_m;\ n(t') = \Gamma\nu\sigma TN(t'),\ n_{\alpha_1}(t') = -\Gamma\nu Tk_{\alpha_1}(t')L_{\alpha_1}/L_m,\ n_{\alpha_1}(t') = -\Gamma\nu Tk_{\alpha_2}(t')L_{\alpha_2}/L_m$ and $n(t') = \beta B\tau S(t')/\nu = \beta\sigma\tau S(t')/\hbar\omega$, the above system can be rewritten as:

$$\begin{aligned}
\frac{dm}{dt'} &= Gm(n + n_{\alpha_1} + n_{\alpha_2} - 1)\\
\frac{dn}{dt'} &= \alpha - n(m+1)\\
\frac{dn_{\alpha_1}}{dt'} &= \delta\alpha_{\alpha_1} - n_{\alpha_1}(\rho_1 m + \delta)\\
\frac{dn_{\alpha_2}}{dt'} &= \delta\alpha_{\alpha_2} - n_{\alpha_2}(\rho_2 m + \delta).
\end{aligned} \tag{2}$$

All the parameters used to define the saturable absorbers are fixed, except for α_{α_1} and α_{α_2}, which include a measure of the active center absorbent density; for this reason, α_{α_1} and α_{α_2} are used as the saturable absorber identifying parameters. Adding an external linear sinusoidal modulation (e.g. using an Electro Optic Modulator (EOM)) directly into the saturable absorbers through their main parameter (i.e. α_{α_1} and α_{α_2}), the last two above equations may be transformed into

$$\begin{aligned}
\frac{dn_{\alpha_1}}{dt'} &= \delta\alpha_{\alpha_1}\left[\frac{1+\cos(\omega_{c_1}t)}{2}\right] - n_{\alpha_1}(\rho_1 m + \delta)\\
\frac{dn_{\alpha_2}}{dt'} &= \delta\alpha_{\alpha_2}\left[\frac{1+\cos(\omega_{c_2}t)}{2}\right] - n_{\alpha_2}(\rho_2 m + \delta),
\end{aligned} \tag{3}$$

where ω_{c_1} and ω_{c_2} stand for the external modulation frequencies applied to the EOM. These four differential equations compose the working system. It must be noted that in absence of a modulation frequency applied to the EOM, ω_{c_1} and ω_{c_2}, the system returns to Eq. (2), i.e. rate equations for a laser with two passive saturable absorbers (Wilson, Aboites, Pisarchik, Pinto, & Barmenkov, 2011).

2. Linear Stability Analysis

Linear Stability Analysis is used to understand the system dynamics (Tabor, 1989; Braun, 1992). The analysis is based on the linear disturbance equations; these equations are derived from the original equations (Pinto Robledo et al., 2012). As it is well known, the method consists in linearizing the describing equations, obtaining the initial state condition (i.e. when the derivatives are zero), expanding the system about the initial state condition, constructing the Jacobian matrix, and finding the eigenvectors and eigenvalues with a determinant equal to zero. This gives, as a result, the fixed points of the equation system, which must be analyzed in order to know what type of points there are (i.e. fixed, source, saddle, etc.) (Wilson, Aboites, Pisarchik, Ruiz-Oliveras, & Taki, 2011; Wilson, Aboites, Pisarchik, Pinto, & Taki, 2011). The equations of interest are Eqs. (2) and (3); these equations are non-autonomous due