Experimental Synchronization of two Integrated Multi-scroll Chaotic Oscillators

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ABSTRACT
Chaotic oscillators have been implemented with a wide variety of discrete electronic devices and quite few realizations using integrated circuit technology. This article describes the synchronization of two chaotic oscillators already fabricated with complementary metal-oxide-semiconductor (CMOS) integrated circuit technology of 0.5um and generating 3- and 5-scrolls. In order to attain the synchronization, we use a master-slave topology with unidirectional coupling. Within this context, a system parameter iterates until the correlation coefficient computed between the chaotic signals generated by the master and slave systems approximates to unity. For the following parameter, its value depends on the standard deviations from the individual signals contrary to previous one. By combining those statistical relationships according to the number of system parameters, we can synchronize integrated chaotic oscillators. Theoretical model simulations of two chaotic oscillators generating 3- and 5-scrolls, and experimental results for two integrated 3-scroll chaotic oscillators validate this approach. Stability and error analysis are also included.

Keywords: Chaos, Synchronization, Integrated Circuit, CMOS, Multi-scroll.

RESUMEN
Los osciladores caóticos se han implementado con una variedad amplia de dispositivos electrónicos discretos y muy pocos con tecnología de circuitos integrados. Este artículo describe la sincronización de dos osciladores caóticos fabricados con tecnología de circuitos integrados CMOS de 0.5um que generan 3- y 5-enrollamientos. Se utiliza la configuración maestro-esclavo para obtener la sincronización. A partir de esta configuración, se itera un parámetro del sistema hasta que el coeficiente de correlación entre las señales caóticas del maestro y el esclavo respectivamente, se aproxima a la unidad. Posteriormente, se calcula la razón de las desviaciones estándar para obtener el valor del siguiente parámetro, esto de forma inversa a la determinación del primero. Es posible sincronizar osciladores caóticos integrados al combinar estas medidas estadísticas en relación al número de parámetros del sistema. Simulaciones del modelo teórico de dos osciladores caóticos exhibiendo tres y cinco enrollamientos, además de resultados experimentales para tres enrollamientos confirman el método propuesto. Son incluidos los análisis de error y estabilidad.

1. Introduction

Carroll and Pecora verified the experimental synchronization of two chaotic oscillators by using operational amplifiers (opamps) [1]. They pointed out that if two independent chaotic systems were started with the same initial conditions, any arbitrarily small perturbation in those conditions would grow exponentially. After some time, the trajectory evolution of the two systems will be uncorrelated. However, if the two chaotic oscillators were synchronized, both systems will evolve to the same chaotic behavior, e.g., in a master-slave topology [2-6], the slave system behaves as the master system despite their chaotic motion, provided they are both driven with a proper signal. In this manner, the general scheme for synchronizing dynamical systems is to take a (nonlinear) system, duplicate some sub-system of that system and drive it with a control signal from the unduplicated part. This is a self-synchronization where the two sub-systems couples by some technique [1-6], [13]. As a function of the synchronized states, we can classify the synchronization approaches as complete synchronization, phase synchronization, lag and intermittent lag synchronization, imperfect phase synchronization, and almost synchronization [14-17, 20-26]. However, those approaches have been generally proved on chaotic systems depicted by either theoretical relationships or electronic circuits designed with discrete devices opposed to the integrated circuit (IC) case [2-6].

Related to multi-scroll chaotic oscillators, they have been implemented using several approaches, such as opamps, operational transconductance amplifiers (OTAs), and current-feedback opamps (CFOAs) [7]. Note that by interconnecting and superimposing unity-gain cells (UGCs) [8-9], one could design those active devices with complementary metal-oxide-semiconductor CMOS IC technology. This approach was previously reported in [10-11], demonstrating that chaotic oscillators can be realized with IC technology. An integrated chaotic oscillator provides key advantages such as; it reduces the form factor (passive and active device count) contrary to the discrete realizations, and the bandwidth of the chaotic signals increases as a function of the time-constants that can be reached with different IC fabrication technologies.

This issue is quite important in an actual encryption scheme based on chaos, which needs high-rate data transmission, as mentioned in [14]. Finally, having ICs one can tune or select the chaotic behavior using a few external passive elements or exploiting the programming capabilities of current mirrors or logic gates. Another advantage in having integrated chaotic oscillators, is that one can develop custom designs that can derive in realizing custom synchronization approaches, thus allowing to realize integrated designs for communication systems, which are in the state-of-the-art [21-24].

In this manner, to the best of our knowledge, this paper is the first one reporting a systematized algorithm to synchronize two chaotic oscillators at integrated circuit level. The novel contribution to the field consists on a systematic algorithm to synchronize integrated chaotic systems with multiple scrolls by using the correlation coefficient (CoCo) and standard deviation (STD) of the numerical time series to reduce the synchronization error iteratively. Besides, the stability conditions are based on conditional Lyapunov exponents computed only once before the first iteration. Therefore, the synchronization is achieved no matter the values of the initial conditions in the synchronization scheme. This approach can be considered as an extension of the complete synchronization technique based on unidirectional coupling [2, 6, 13].

The main idea of this method consists on sending a chaotic signal from the integrated nonlinear system (master) to another (slave). Then, the slaved system tracks the dynamics of the master system. This means that the behavior of the second system does not have any influence on the first one. Additionally, a 45-degree straight line, on the phase space portrait, is a well-accepted criterion to confirm the synchronization. This results when the same state-variables taken from the integrated master and slave dynamical systems are compared. Note that the dynamical evolution of the chaotic signals is theoretically identical after the synchronization is attained. Within this context, we computed CoCo between the time series of both the master and slave systems. When the data are correlated, the synchronization can be ensured. In a similar way, we used STD of independent chaotic signals to verify the synchronization error, i.e., when this measure converges to the same value implies that chaotic signals are close related and synchronized. Additionally, we can adjust a positive lineal approximation for both statistical measures by varying the parameters in the slave system. It is also proved that this approach is asymptotically stable by computing the conditional Lyapunov exponents. Several numerical simulation results for 3- and 5-scroll chaotic attractors confirm the synchronization approach. Finally, experimental