Ranking of Exponential Vague Sets With an Application to Decision Making Problems

Pushpinder Singh

Department of Computer Science,
Palacky University, 17. listopadu 12, CZ-77146,
Olomouc, Czech Republic.
pushpindersnl@gmail.com

ABSTRACT
The main aim of this paper is to propose a new approach for the ranking of exponential vague sets. The concepts of exponential vague sets and arithmetic operations between two exponential vague sets are introduced. The shortcomings of some existing ranking approaches for the ranking of generalized fuzzy sets and intuitionistic fuzzy numbers are pointed out. The proposed method consider not only the rank but also the decision maker optimistic attitude and it is shown that proposed ranking approach is more intuitive and reasonable as compared to existing ranking approaches. Also the proposed ranking function satisfies the reasonable properties for the ordering of fuzzy quantity. For practical use, proposed ranking approach is applied to decision making problem.

Keywords: Fuzzy sets, vague sets, exponential vague sets, intuitionistic fuzzy numbers, ranking functions, decision making problems.

RESUMEN
El objetivo principal de este trabajo es proponer un nuevo enfoque para la clasificación de los conjuntos inciertos exponenciales. Se introducen los conceptos de conjuntos inciertos exponenciales y operaciones aritméticas entre dos conjuntos inciertos exponenciales. Se señalan las deficiencias de algunos enfoques de clasificación existentes para la clasificación de los conjuntos difusos generalizados y de los números difusos intuicionistas. El método propuesto toma en cuenta no sólo el rango, sino también el enfoque optimista para toma de decisiones y se muestra que el enfoque de clasificación propuesto es más intuitivo y razonable en comparación con los enfoques de clasificación existentes. Asimismo, la función de clasificación propuesta satisface las propiedades razonables para el ordenamiento de la cantidad difusa. Para usos prácticos, el enfoque de clasificación propuesto se aplica al problema de toma de decisiones.

1. Introduction

The theory of fuzzy sets was first introduced by Zadeh [37] in 1965. Since then, the theory of fuzzy sets is applied in many fields such as pattern recognition, control theory, management sciences and picture processing, etc. In the field of fuzzy mathematics many mathematical theory such as fuzzy optimization, fuzzy topology, fuzzy logic, fuzzy analysis and fuzzy algebra etc. are obtained [3, 10, 14, 22, 23, 29, 32, 35]. In many applications of fuzzy set theory to decision making, we are faced with the problem of selecting one from a collection of possible solution, and in general we want to know which one is the best. This selection process may require that we rank or order fuzzy numbers. In order to rank fuzzy numbers, one fuzzy number needs to be evaluated and compared to others but this may not be easy. As known, the real numbers in can be linearly ordered by, however, fuzzy numbers cannot be done in such a way. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than the other.

To the task of comparing fuzzy numbers, many authors proposed fuzzy ranking methods [5, 6, 8, 9, 12, 15, 16, 17, 18, 20, 25, 30, 31, 34]. But among all the methods, most of them consider only one point of view on comparing fuzzy quantities in spite of the different demand of the decision maker, so some improved methods have been brought forward which lead to produce different rankings for the same problem. Until now, there have not one unify method to this problem. Fuzzy set theory [37] has been shown to be useful tool to handle the situations, in which the data is imprecise, by attributing a degree to which a certain object belongs to a set. In real life, a person may assume that an object belongs to a set, but it is possible that he is not sure about it. In other words, there may be hesitation or uncertainty thatwhether an object belongs to a set or not. In fuzzy set theory, there is no means to incorporate such type of hesitation or uncertainty. A possible solution is to use intuitionistic fuzzy set [2] and vague set [11]. Bustince and Burillo [4] pointed out that the notion of vague set is the same as that of intuitionistic fuzzy set. Lu and Ng [21] proved that vague sets is more natural than using an intuitionistic fuzzy set. Several authors [19, 24, 27, 28] have proposed different methods for the ranking of intuitionistic fuzzy sets but to the best of our knowledge till now there no method in the literature for the ranking of vague sets.

The main aim of this paper is to propose a new approach for the ranking of exponential vague sets. The concepts of exponential vague sets and arithmetic operations between two exponential vague sets are introduced. The shortcomings of some existing ranking approaches [8, 19] for the ranking of generalized fuzzy sets and intuitionistic fuzzy numbers are pointed out. Also it is shown that proposed ranking approach is more intuitive and reasonable as compared to existing ranking approaches Rest of the paper is organized as follows: In Section 2, some basic definitions related to generalized fuzzy sets, intuitionistic fuzzy sets, vague sets and arithmetic operations between vague sets are presented. In Section 3, a brief review of the existing approach [8] for the ranking of generalized trapezoidal fuzzy numbers and the existing approach [19] for the ranking of intuitionistic fuzzy numbers are presented. In Section 4, the shortcomings of existing approaches [8, 19] are discussed. In Section 5, a new approach is proposed for the ranking of exponential vague sets. In Section 6, it is proved that the proposed ranking function satisfies the reasonable properties for the ordering of fuzzy quantities and results are compared with some existing approached. In Section 7, an application of proposed ranking method to decision making is presented. Section 8 draws the conclusions.

2. Preliminaries

In this section some basic definitions related to generalized fuzzy sets, intuitionistic fuzzy sets, vague sets and arithmetic operations between exponential vague sets are presented.

2.1 Generalized Fuzzy Sets

Definition 1. [6] A fuzzy set A, defined on the universal set of real numbers R, is said to be a generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_A : R \to [0, w]$ is continuous.

2. $\mu_A(x) = 0$, for all $x \in (-\infty, a] \cup [d, \infty)$.

3. $\mu_A(x)$ is strictly on $[a,b]$ and strictly decreasing on $[c,d]$.

4. $\mu_A(x) = w$, for all $x \in [b,c]$, where $0 < w \le 1$.

Definition 2. [6] A generalized fuzzy number, denoted as $A = (a, b.c, d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_A(x) = \begin{cases} \dfrac{w(x-a)}{(b-a)}, a \le x < b, \\ w \quad ,b \le x \le c, \\ \dfrac{w(x-d)}{(c-d)}, c < x \le d, \\ 0 \quad , \text{otherwise} \end{cases}$$

2.2 Intuitionisitic Fuzzy Sets

Definition 3. [19] An intuitionistic fuzzy set $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$ on the universal set X is characterized by a truth membership function $\mu_A(x), \mu_A(x) : X \to [0,1]$ and a false membership $\nu_A(x), \nu_A(x) : X \to [0,1]$. The values $\mu_A(x)$ and $\nu_A(x)$ represents the degree of membership and degree of non-membership of x and always satisfies the condition $\mu_A(x) + \nu_A \le 1$. The value $1 - \mu_A(x) + \nu_A$ represents the degree of hesitation of $x \in X$.