On Feedback Control Techniques of Nonlinear Analytic Systems

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ABSTRACT
This paper presents three approaches dealing with the feedback control of nonlinear analytic systems. The first one treats the optimal control resolution for a control-affine nonlinear system using the State Dependant Riccati Equation (SDRE) method. It aims to solve a nonlinear optimal control problem through a Riccati equation that depends on the state. The second approach treats a procedure of constructing an analytic expression of a nonlinear state feedback solution of an optimal regulation problem with the help of Kronecker tensor notations. The third one deals with the global asymptotic stabilization of the nonlinear polynomial systems. The designed state feedback control law stabilizes quadratically the studied systems. Our main contribution in this paper is to carry out a stability analysis for this kind of systems and to develop new sufficient conditions of stability. A numerical-simulation-based comparison of the three methods is finally performed.

Keywords: Nonlinear systems, Optimal control, State Dependant Riccati Equation (SDRE), Feedback control, stability analysis.

1. Introduction

The optimal control of nonlinear systems is one of the most challenging and difficult topics in control theory. It is well known that the classical optimal control problems can be characterized in terms of Hamilton-Jacobi Equations (HJE) [1, 2, 3, 4, 5, 6]. The solution to the HJE gives the optimal performance value function and determines an optimal control under some smooth assumptions, but in most cases it is impossible to solve it analytically. However, and despite recent advances, many unresolved problems are steel subsisting, so that practitioners often complain about the inapplicability of contemporary theories. For example, most of the developed techniques have very limited applicability because of the strong conditions imposed on the system [29, 40, 41, 42, 43]. This has led to many methods being proposed in the literature for ways to obtain a suboptimal feedback control for general nonlinear dynamical systems.

The State Dependent Riccati Equation (SDRE) controller design is widely studied in the literature as a practical approach for nonlinear control problems. This method was first proposed by Pearson in [7] and later expanded by Wernli and Cook in [8]. It was also independently studied by Cloutier and all in [9, 31, 34]. This approach provides a very effective algorithm for synthesizing nonlinear optimal feedback control which is closely related to the classical linear quadratic regulator. The SDRE control algorithm relies on the solution of a continuous-time Riccati equation at each time update. In fact, its strategy is based on representing a nonlinear system dynamics in a way to resemble linear structure having state-dependant coefficient (SDC) matrices, and minimizing a nonlinear performance index having a quadratic-like structure [9, 24, 34, 35, 38]. This makes the equation much more difficult to solve. An algebraic Riccati equation using the SDC matrices is then solved on-line to give the suboptimum control law. The coefficients of this equation vary with the given point in state space. The algorithm thus involves solving, at a given point in state space, an algebraic state-dependant Riccati equation, or SDRE.

Although the stability of the resulting closed loop system has not yet been proved theoretically for all

system kinds, simulation studies have shown that the method can often lead to suitable control laws.
Due to its computational simplicity and its satisfactory simulation/experimental results, SDRE optimal control technique becomes an attractive control approach for a class of non linear systems. A wide variety of nonlinear control applications using the SDRE techniques are exposed in literature. These include a double inverted pendulum in real time [26], robotics [12], ducted fan control [37, 38], the problems of optimal flight trajectory for aircraft and space vehicles [22, 30, 32, 36] and even biological systems [10, 11].

An other efficient method to obtain suboptimal feedback control for nonlinear dynamic systems was firstly proposed by Rotella [33]. A useful notation was developed, based on Kronecker product properties which allows algebraic manipulations in a general form of nonlinear systems. To employ this notation, we assume that the studied nonlinear system is described by an analytical state space equation in order to be transformed in a polynomial modeling with expansion approximation. In recent years, there have been many studies in the field of polynomial systems especially to describe the dynamical behavior of a large set of processes as electrical machines, power systems and robot manipulators [14, 15, 16, 17, 18]. A lot of work on nonlinear polynomial systems have considered the global and local asymptotic stability study, and many sufficient conditions are defined and developed in this way [14, 15, 19, 20, 21, 39].

The present paper focuses on the description and the comparison of three nonlinear regulators for solving nonlinear feedback control problems: the SDRE technique, an optimal regulation problem for analytic nonlinear systems (presented for the first time by Rotella in [33]) and a quadratic stability control approach. A stability analysis study is as well carried out and new stability sufficient conditions are developed.

The rest of the paper is organized as follows: the second part is reserved to the description of the studied systems and the formulation of the nonlinear optimal control problem. Then, the third part is devoted to the presentation of approaches of the optimal control resolution and quadratic stability control approach, as well as to the illustration of sufficient conditions for the existence of solutions to the nonlinear optimal control problem, in particular by SDRE feedback control. In section 4 we give the simulation results for the comparison of the three feedback control techniques. Finally conclusions are drawn.

2. Description of the studied systems and problem formulation

We consider an input affine nonlinear continuous system described by the following state space representation:

$$\begin{cases} \dot{X} &= f(X) + g(X)U \\ Y &= h(X) \end{cases} \tag{1}$$

with associated performance index:

$$J = \frac{1}{2}\int_0^\infty (X^T Q(X) X + U^T R(X) U)dt \tag{2}$$

where $f(X)$, $g(X)$ and $h(X)$ are nonlinear functions of the state $X \in \mathrm{R}^n$, U is the control input and the origin (X=0) is the equilibrium, i.e $f(0)=0$.

The state and input weighting matrices are assumed state dependant such that: $Q:\mathrm{R}^n \to \mathrm{R}^{n\times n}$ and $R:\mathrm{R}^n \to \mathrm{R}^{m\times m}$. These design parameters satisfy Q(X)>0 and R(X)>0 for all X.

The problem can now be formulated as a minimization problem associated with the performance index in equation (2):

$$\min_{U(t)} \frac{1}{2}\int_0^\infty (X^T Q(X) X + U^T R(X) U)dt \quad \text{subject to } \dot{\mathrm{X}} = \mathrm{f(X)} + \mathrm{g(X)U}, \quad \mathrm{X}(0) = X_0 \tag{3}$$

The solution of this nonlinear optimal control problem is equivalent to solving an associated Hamilton-Jacobi equations (HJE) [1].

For the simpler linear problem, where $f(X)=A_0X$, the optimal feedback control is given by $U(X) = -R^{-1}B^T PX$, with P solving the algebraic Riccati equation $PA_0 + A_0^T P - PBR^{-1}B^T P + Q = 0$.

The theories for this linear quadratic regulator (LQR) problem have been established for both the