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Time-dependent interaction across two conducting spheres in an applied electrostatic field

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ABSTRACT

Charged particles exist widely in variety of technological areas as well as in nature. Even a weak charge on the particles can significantly influence their electric interaction. We investigated the phenomenon of time-dependent electric interaction between two conducting spheres in an electrostatic field. A mirror-image method was developed to analyze this system, and the fundamental role of the charges on the spheres was studied. We concluded that charges conducted to the lower sphere through the alumina tube used in our system play a main role in determining the time-dependent interaction, whereas the influence from air ions is negligible.

1. Introduction

The interaction between dielectric or conducting spheres in applied electric fields has been researched widely because of their applications in electrorheological fluids [1–4], electrophoresis [5,6], and other fields [7–9]. The interaction between two spheres induced by an applied electric field has also been the subject of many fundamental investigations [10–16]. A variety of theoretical models, such as the finite-element method [14], dipole approximation [17], and multipole-expansion theory [18–21] can be used to describe this two-sphere system. Although results from experiments agree with the theoretical calculations with acceptable precision, in most experiments, an alternating current (AC) electromagnetic field is used [9,10,12,15]. As discussed by Tao and Lan [15], the accumulation of ions in the medium surrounding the spheres affects the interaction. In general, AC electromagnetic fields are used to avoid the influence from surrounding ions [9,10,12,15]. However, interactions in electrostatic fields are very important because in many areas of research and engineering, a direct current (DC) electromagnetic field is used. For example, some work on electrorheological fluids uses a DC electromagnetic field.

When charged spheres are subjected to an applied electric field, the interaction induced is not only determined by the applied electric field, but also by the charges on the spheres. Moreover, the charge distribution is influenced by both the applied electric field and the electric field induced by the charges on the spheres. The mirror-image method (hereafter, ‘image method’) is a powerful means to calculate the interaction between conducting spheres. This method was first developed in the 19th century by Thomson to treat simple electrostatic problems in symmetrical systems [22] and has subsequently been used for many problems involving conducting systems [23–25]; in particular, it has been widely applied for systems with two conducting spheres [26–32]. In most cases, the image method is used for systems without free charges. Van den Bosch et al. [33] and Jones [34] discussed systems with charges, but used imaginary charges instead of the original potentials of the spheres. Gao et al. [35] pointed out that the electric interaction between dielectric spheres with high permittivity is almost identical to that between conducting spheres; thus, it is also feasible to study dielectric spheres with high permittivity using the image method.

Electrorheological fluids are always taken as system with dielectric particles dispersed in insulative liquid [4]. Though limited conductivity exists in these materials, its influence was always neglected in the past. In order to clarify the influence of charge transfer and accumulation, we conducted an experiment to measure the interaction between two conducting spheres in an applied electrostatic field. The image method was used to calculate the

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interaction between two spheres with different charges in an applied electrostatic field. We compared the calculated results with those from the experiment to analyze the magnitude and polarity of the charges on the spheres.

2. Experimental details

The apparatus used in this experiment, as shown in Fig. 1, consisted of two horizontal fixed copper plates (160.0 mm in diameter) separated by a distance of 25.0 mm with Teflon poles. Two titanium spheres (6.3 mm in diameter) were separated in the space between the two copper plates. A stepper motor controlled by a computer was used to adjust the distance between the two spheres. An electronic balance with 1.0-mg precision and a grating ruler with 0.1-μm precision were connected to a computer to record the force and displacement. The lower sphere was fixed on the lower electrode plate by an insulating alumina tube with a small diameter of 1.2 mm; the upper sphere was fixed to the other alumina tube and hung from the hook under the balance through a small aperture in the center of the upper electrode plane.

The connecting line of the sphere centers was first adjusted to be vertical by four screw micrometers in different directions, and the stepper motor was lifted to make the two spheres touch exactly. In this way, the gap between spheres is the displacement of the stepper motor when it is lowered under the control of the computer. The force applied to the upper sphere could be measured by the balance. A high-voltage DC power supply was used, the anode and cathode of which were connected to the upper and lower electrodes, respectively. Thus, a homogeneous electric field could be produced in the space between the two plate electrodes. The direction of the electric field in this study was always downward. The two spheres were assumed to be isolated. When the DC voltage is fixed, the two spheres will attract each other because of electric induction. We measured this attraction with the electronic balance and recorded the data on a computer using software written by us. The instrument was sealed in a transparent box to avoid the influence of air flow. The ambient gas in the box was atmospheric air, the room temperature was kept at 24 °C, and the relative humidity in the box was 60% ± 5%.

3. Calculation

When a point charge or a dipole moment is placed near a conducting sphere, the electric field in the space outside the sphere can be assumed to be the combination of the charge or dipole moment and their images in the sphere [34–38]. For a point charge Q_0, its image charge in a sphere with radius R is defined as

$$Q = -\frac{R}{d}Q_0, \tag{1}$$

where d is the distance between the point charge and the center of the sphere.

The distance between the position of the image charge Q and the center of the sphere is defined as

$$r = \frac{R^2}{d}, \tag{2}$$

To keep the sphere neutral, another compensatory point charge, Q', at the center of the sphere is needed, which is defined as

$$Q' = -Q. \tag{3}$$

The images of the dipole moment $\mathbf{p}_0$ include a dipole moment, $\mathbf{p}$, a point charge, q, and a compensatory point charge, q', at the center of the sphere; these are expressed as

$$\mathbf{p} = \left(\frac{R}{d}\right)^3 \mathbf{p}_0, \tag{4}$$

$$q = -\frac{R}{d^2}p, \tag{5}$$

$$q' = -q = \frac{R}{d^2}p. \tag{6}$$

The positions of $\mathbf{p}$ and q are the same as in Equation (2), and the position of q' is same as that of Q'.

When two conducting spheres A and B separated with a small distance d are placed in an applied homogeneous electric field $\mathbf{E}_0$ oriented parallel to the line connecting the centers of the spheres, the initial electric induction can be taken as two dipole moments located at the centers of the spheres. The dipole moment in one sphere will induce a series of images in the other sphere, and vice versa. If the two spheres are first charged with Q_A and Q_B, the charges should be located at the center of each sphere. Similarly, the charge in one sphere will induce a series of images in the other sphere. The initial dipole moments and point charges can be calculated as the first-order images:

$$\mathbf{p}_{A,1} = \mathbf{p}_{B,1} = 4\pi\varepsilon R^3 \mathbf{E}_0, \tag{7}$$

$$q_{A,1} = Q_A, \tag{8}$$

$$q_{B,1} = Q_B, \tag{9}$$

the positions of which are at the centers of the two spheres. The higher order images can be written as

$$\mathbf{p}_{A,i} = \left(\frac{R}{d - r_{B,i-1}}\right)^3 \mathbf{p}_{B,i-1}, \tag{10}$$

$$q_{A,j} = -\frac{R}{(d - r_{B,j-1})^2} p_{B,j-1} - \frac{R}{d - r_{B,j-1}} q_{B,j-1}, \quad j = i, i-1, \ldots, 2 \tag{11}$$

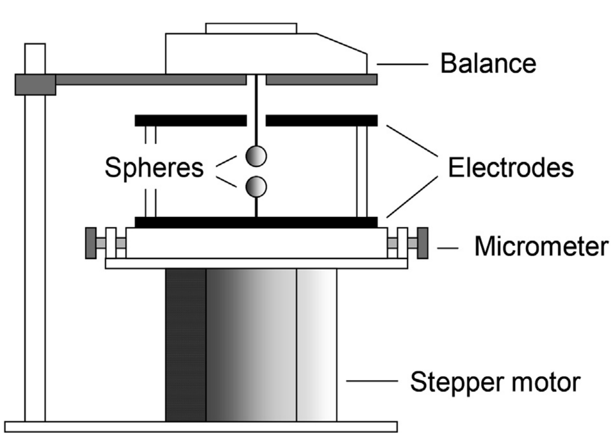

Fig. 1. Sketch of the apparatus used in the experiment.