Journal of Electrostatics 67 (2009) 751–758

Contents lists available at ScienceDirect

Journal of Electrostatics

journal homepage: www.elsevier.com/locate/elstat

Study of dielectric behaviour of woven fabric based on two phase models

Kausik Bal, V.K. Kothari*

Department of Textile Technology, Indian Institute of Technology, 8 New Campus, Hauz Khas, New Delhi, Delhi 110016, India

ARTICLE INFO

Article history:
Received 29 November 2008
Received in revised form
13 March 2009
Accepted 7 May 2009
Available online 12 June 2009

Keywords:
Capacitance
Dielectric constant
Fibre
HDPE
Mathematical model
Monofilaments
Peirce's fabric geometry
Textile
Woven fabric

ABSTRACT

Dielectric constant of textile fibres plays very important role in electrostatic behavior of textile materials during its processing and use. The effective dielectric constant of textile materials is defined as the ratio of capacitance of a parallel plate capacitor with the textile material to that of the capacitor without the textile material. This paper presents three models considering textile woven fabric as a mixture of air and fibre to relate dielectric constant of fibre material and the effective dielectric constant of fabric. The mathematical models have taken into account measured fabric parameters. Plain woven fabrics of high density polyethylene monofilament yarns were used to do the actual measurement and results of three models based on these fabrics are compared.

1. Introduction

Theoretical studies on dielectric behaviour of textile substrates kept between two parallel capacitor plates have been done by many researchers in the past. Some of these studies considered the behaviour of a fibrous mass under an electric field without considering any particular textile structure. Morton and Hearle [1] has discussed various such models which were empirically proposed to calculate the dielectric constant of fibres from capacitance of a fibrous mass of a given porosity. Boyd [2] developed an equation to calculate the change in capacitance of a parallel plate capacitor when a straight yarn is passed through the space between the plates. His equation had the yarn porosity, fibre density and dielectric constant of the fibre material as material parameters which could be effectively used to establish a mathematical relationship between the change in capacitance Δc and the linear density of the yarn. Mack [3] has investigated the factors that determine the behaviour of a cylindrical dielectric such as a textile fibre or yarn when kept between a parallel plate capacitor. Recently, Alekseeva [4] has derived equation relating the capacitance and dielectric constant of textile fibre material in a solid or hollow cylindrical shape. However, theoretical work on the dielectric behaviour of fabrics seems to be a rare thing in literature. The present paper proposes three models to establish a relationship between the dielectric constant of the fibre material and the capacitance of a parallel plate capacitor with the woven fabric between the parallel plates. The ratio of the capacitance of a parallel plate capacitor with a uniform homogeneous dielectric material occupying the entire space between the plates to that of the same capacitor system with air or vacuum occupying the entire volume between the plates is known as the dielectric constant or relative dielectric permittivity of the dielectric material. In case of a heterogeneous material such as a textile fabric material which is a mixture of air and fibres, the same ratio can be called as effective dielectric constant of the air-fibre mixture. For the sake of simplicity, the fabric has been considered to be consisting of only fibrous polymers and air, i.e., the effect of moisture has not been considered. Hence the present work is only limited to models having only two phases: fibres and air. The basic approach has been to incorporate the measurable fabric parameters in the model in order to make the theory useful.

Three mathematical models have been discussed and proposed in this relation in this paper starting from an equivalent sheet which resembles the existing approaches that relates the dielectric

* Corresponding author. Tel.: +91 11 2659 1407; fax: +91 11 2658 1103.
E-mail address: kotharivk@gmail.com (V.K. Kothari).

0304-3886/$ – see front matter
doi:10.1016/j.elstat.2009.05.005

constant of fibre material with the effective dielectric constant of fibrous mass. The next model considers the fabric as an orthogonal set of yarns in two layers resembling a non-crimped fabric and the last model considers a woven fabric following Peirce's simple fabric geometry [5].

Woven fabrics generally consist of two sets of yarns lying orthogonal to each other. One set, known as warps, consists of a number of parallel yarns running along the length of fabric and the other set, known as wefts, consists of a number of parallel yarns running along the width of the fabric. In non-crimped fabric, these two set of yarns remain in two parallel planes one above another and never intersect. However, in most of the traditional woven fabrics, the warp and weft yarns get interlaced at the points where they cross over. In plain woven fabric, such interlacement follows a typical pattern such that any given yarn of warp or weft alternately remains on top and bottom of the yarns of the other set throughout the fabric. This causes the yarns to have a waviness which is called 'crimp'. The construction of a woven fabric is determined by several parameters some of which are considered here. One such parameter is the number of warp yarns per unit width, called 'warp yarn thread density' expressed as 'ends per metre', an end being a warp yarn. The gap between two successive warp yarns is called 'warp yarn spacing'. The extent of waviness of warp yarn is denoted by 'crimp' which is measured by the ratio of the difference of the length of a given straight warp yarn and the length of the same yarn in fabric to the length of the same warp yarn in fabric, the length of the yarn in fabric naturally being smaller due to waviness, expressed in percentage. The maximum angle that such a crimped warp yarn makes to the plane of the fabric is called weave angle for warp yarn. The other parameters such as 'weft yarn thread density' (which is expressed as 'picks per metre', a pick being a weft yarn), weft yarn spacing, weft crimp and weave angle for weft yarn are the corresponding terms for the weft yarns respectively.

Results of the three models have been compared with respect to capacitance data using monofilament woven fabric of high density polyethylene (HDPE).

The models discussed in this paper may be applied in case of fabric or fibre reinforced composites as far as dielectric characterization is concerned (such as fabric reinforced PCBs), filtration fabrics where the fibres could be electrostatically charged, identification and characterization of fibres when measurement is done in a fabric form, studies related to ESD regarding textile fabrics, etc.

2. Models for calculation of dielectric constant

Considering the dielectric medium between the two parallel electrode plates as a mixture of air and fabric, three mathematical models, each assuming a different simplified geometry of the fabric from the other, are considered. The models are discussed below.

2.1. First model: equivalent solid film

It is assumed that the fabric can be replaced by a uniform homogeneous polymeric film having same dielectric constant, density and volume as the fibre having an appropriate thickness, without affecting the overall capacitance of the system.

Let

Area of parallel plate capacitor $= A$ (m^2)
Gap between parallel plates $= L$ (m)
Fabric areal density $= G$ (g/m^2) $= 0.001 \times G$ (g/m^2)
Fibre density $= \rho$ (Kg/m^3)
Dielectric constant of fibre polymer $= K$
Thickness of equivalent polymer film $= h$ (m)
The volume of fibrous material in the capacitor $= (0.001 \times G \times A)/\rho$ (m^3)

This is equal to the volume of the equivalent polymer film as per the assumption, hence,

$$h \times A = \frac{0.001 \times G \times A}{\rho}$$

or

$$h = \frac{0.001 \times G}{\rho} \tag{1}$$

The total capacitance of the system is a series connection of capacitance of the electrodes filled with equivalent polymer film and having the same gap as the thickness of this film and the capacitance of the electrodes filled with air and having a gap equal to the difference between the actual electrode gap and the effective thickness of the equivalent polymer film. Assuming the dielectric constant of air as unity, the total capacitance can be calculated as

$$C = \frac{\epsilon_0 \times A}{L - \left(\frac{0.001 \times G}{\rho}\right) \times \left(1 - \frac{1}{K}\right)} \tag{2}$$

Where ϵ_0 is permittivity of free space ($\epsilon_0 = 8.854$ pF/m).

2.2. Second model: consideration of simplified mesh of cylindrical yarns

In this case it is assumed that the fabric is made of two transversely arranged set of yarns which are laid in two layers without any interlacement. The yarn cross-section is assumed to be circular. A simple algebraic treatment is not suitable in such case; hence a different mathematical tool will be adopted. First, the more general problem of calculating the capacitance of an arbitrary volume of material having any given shape will be considered.

With reference to Fig. 1, the capacitance of an elementary area of the capacitor having a dielectric material of arbitrary shape in between its parallel plates is given as

$$dc = \epsilon_0 \times \frac{dxdy}{\frac{L-u}{1.0} + \frac{u}{K}} = \frac{\epsilon_0}{L} \times \frac{dxdy}{\left\{1 - \left(1 - \frac{1}{K}\right) \times \frac{u}{L}\right\}}$$

Hence, total capacitance is

$$C = \frac{\epsilon_0}{L} \times \int_{y_1}^{y_2} \int_{x_1}^{x_2} \frac{dxdy}{\left\{1 - \left(1 - \frac{1}{K}\right) \times \frac{u}{L}\right\}} \tag{3}$$

Where L is the gap between the parallel plates, $(x_2 - x_1)$ is the length of parallel plate along X axis and $(y_2 - y_1)$ is the width of the parallel plate along Y axis and u is the total length of material along Z axis at a point (x, y). Here, the origin is assumed to be at the centre of the bottom plate.

Applying this to the case of a cylindrical yarn, as shown in Fig. 2,

$$u_i = 2 \times \sqrt{\left(\frac{d_1}{2}\right)^2 - x^2} \tag{4}$$