Journal of Electrostatics 78 (2015) 60–67

Contents lists available at ScienceDirect

Journal of Electrostatics

journal homepage: www.elsevier.com/locate/elstat

EHD convection in dielectric micropolar fluid layer

Neela Rani [a], S.K. Tomar [b,*]

[a] Department of Mathematics, MCM DAV College, Chandigarh, 160 036, India
[b] Department of Mathematics, Panjab University, Chandigarh, 160 014, India

ARTICLE INFO

Article history:
Received 11 July 2015
Received in revised form
14 October 2015
Accepted 29 October 2015
Available online 15 November 2015

Keywords:
Instability
Micropolar viscosity
Convection
Electric field
Rayleigh number
Dielectric fluid

ABSTRACT

The onset of instability in a layer of dielectric micropolar fluid under the simultaneous action of an *AC* electric field and temperature gradient has been investigated. The dispersion relation has been derived and various critical values of non-dimensional Rayleigh number in the fluid layer have been determined. The influence of micropolar viscosity and electric Rayleigh number on the onset of convection has been analyzed. Thermal Rayleigh number has been computed for various values of electric Rayleigh number for the onset of instability. The stabilizing and destabilizing effects of electric Rayleigh number, micropolar viscosity and Prandtl number have been discussed.

1. Introduction

Electro-hydrodynamics is concerned with the mechanics of fluid flow under the interactions of externally applied electric field. The externally applied electric field on the fluid motion generates an instability phenomena in microchannels. It has been noticed that using electric force in the motion of fluids is a very effective method in getting a variety of results and functions. These results are very helpful in the application of micro-fluidic devices which are widely used now a days. The applications of electro-hydrodynamic transport phenomena may be found in the area of mechanical engineering, where it is used in devices like electrokinetic assays, electro-spray ionization, electro-coalescence and mixing, electrostatic printing and spinning. The instability of flow is required in certain applications such as in mixing, while stable flow is typically the preferred state (e.g. in assays and ionization) may be required in some other devices. Thus the demarcation between the stable and unstable states is of great practical importance.

A temperature gradient applied to a dielectric fluid produces a gradient in the dielectric constant and electrical conductivity. Keeping this fact in mind, several problems of the onset of convection instability in a horizontal layer of dielectric fluid under the action of a vertical *DC* electric field and a vertical temperature gradient have been investigated in the past. Notable among them are Gross and Porter [1], Gross [2], Gelmont and Ioffe [3], Turnbull [4,5], Roberts [6], Takashima and Aldridge [7] and Lin [8]. The application of a *DC* electric field results in the accumulation of free charge in the fluid. The free charge so build up occurs exponentially in time with a time constant ε/σ, where ε is the dielectric constant and σ is the electrical conductivity. This time constant is known as the *electrical relaxation time*. If instead of a *DC* electric field, an *AC* electric field is applied at a frequency which is much higher than the reciprocal of the electrical relaxation time, then the free charge cannot accumulate due to the rapid movement. The electrical relaxation time of most dielectric fluids appear to be sufficiently long to make free charge effects negligible at standard power line frequencies. In such cases, the dielectric loss is too low to make any effective contribution in the temperature field. At this very moment, variations in the body force are so rapid that its mean value is taken as the effective value in determining fluid motions, except in the case of fluids of extremely low viscosity. Therefore, the case of *AC* electric field is easy to manage as compared to the case of *DC* electric field. The effect of *AC* and/or *DC* electric field in the onset of convection in a dielectric fluid layer play very important role and has been investigated by several researchers in the past, e.g., Roberts [6], Jones [9], Meakawa et al. [10], Saville [11], Shivkumara et al. [12] including several others. Analysis of electro-hydrodynamic

* Corresponding author.
E-mail addresses: neela.pawar@yahoo.com (N. Rani), sktomar66@gmail.com (S.K. Tomar).

http://dx.doi.org/10.1016/j.elstat.2015.10.003
0304-3886/

instability in a horizontal fluid layer with electrical conductivity gradient subjected to a weak shear flow has been discussed by Chang et al. [13]. Recently, a study of rotating couple stress dielectric fluid layer has been attempted by Shivkumara et al. [18] and electro convection in a micropolar fluid under the effect on non-uniform temperature gradient has been investigated by Pranesh and Baby [19].

Many problems of thermal convection based on Chandrasekhar model [15] in a horizontal layer of Newtonian fluid heated from below under varying assumptions of hydrodynamics and hydromagnetics have been studied in the past. One of the main objective in all these problems is to determine the critical Rayleigh number and to study the effect of various parameters incorporated on the onset of convection in the fluid layer. It is doubtless to advocate that the fluid characteristics (e.g., suspended particles, salinity, nanoparticles etc) do affect the onset of convection and play very important role in better understanding the entire convection phenomena. Keeping in view the importance of fluid filaments, we aimed to study the problem of onset of instability in a horizontal layer of micropolar fluid under the action of *AC* electric field and temperature gradient. The theory of micropolar fluid introduced earlier by Eringen [14] has been employed, in which the fluid particles can undergo micro-rotation, in addition to translation. Colloidal fluids, physiological fluids, cellular fluid, fluid coming out as industrial waste, some paint like fluids etc lie within the category of micropolar fluids. The instability of micropolar fluid under the effect of various parameters has been extensively studied by many authors. The reference of Datta and Sastry [16] and Siddheshwar and Pranesh [17] are worth notable.

2. Mathematical formulation

Consider a layer of incompressible dielectric micropolar fluid having thickness d and of infinite extent. Referred to cartesian coordinate system (x, y, z), we take the origin at the bottom of the layer and z-axis normal to the fluid layer in the gravitational field. Thus the fluid layer is bounded below and above by the planes $z=0$ and $z=d$ respectively. Both the lower and upper surfaces are maintained at constant temperatures T_0 and $T_1(<T_0)$ respectively. In addition to the temperature gradient, a uniform vertical *AC* electric field has also been imposed across the layer. The lower surface is grounded, while the upper surface has been kept at an alternating (60 *Hz*) potential whose root mean square value is ϕ.

Following Eringen [14] and Landau and Lifshitz [20], the relevant basic equations governing the system in the absence of body couples are given as

$$\nabla\cdot\mathbf{v}=0, \tag{1}$$

$$\rho_0\frac{D\mathbf{v}}{Dt}=-\nabla p+\rho\mathbf{g}+(\mu+\kappa)\nabla^2\mathbf{v}+\kappa\nabla\times\mathbf{\Pi}+\mathbf{f}_e, \tag{2}$$

$$\rho_0 j\frac{D\mathbf{\Pi}}{Dt}=(\varepsilon+\beta')\nabla(\nabla\cdot\mathbf{\Pi})+\gamma'\nabla^2\mathbf{\Pi}+\kappa(\nabla\times\mathbf{v}-2\mathbf{\Pi}), \tag{3}$$

$$\rho_0 c_v\frac{DT}{Dt}=k_t\nabla^2T+\delta(\nabla\times\mathbf{\Pi})\cdot\nabla T, \tag{4}$$

$$\nabla\cdot(\varepsilon\mathbf{E})=0, \tag{5}$$

$$\nabla\times\mathbf{E}=0, \tag{6}$$

where $\mathbf{v}=(u,v,w)$, $\mathbf{\Pi}$, $\mathbf{g}=(0,0,-g)$, ρ, p, j, c_v, k_t, T, δ, ε, $\mathbf{E}=(0,0,E_z)$ represent respectively velocity, spin, gravitational acceleration, density, pressure, micro-inertia, specific heat at constant volume, thermal conductivity, temperature, coupling coefficient between heat flux and spin flux, electric constant and electric field. Equation (6) gives that $\mathbf{E}=-\nabla\phi$, where ϕ is the root mean square of the electric potential. The quantities ε, β', γ', and κ are the micropolar fluid viscosities and

$$\frac{D}{Dt}\equiv\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla,$$

is the material derivative. The last term in Equation (2), namely, $\mathbf{f}_e$ is the force of electric origin given by (see Landau and Lifshitz [20], pp-68)

$$\mathbf{f}_e=\rho_e\mathbf{E}-\frac{1}{2}E^2\nabla\varepsilon+\frac{1}{2}\nabla\left(\rho\frac{\partial\varepsilon}{\partial\rho}E^2\right). \tag{7}$$

In our analysis, we shall neglect the first term, namely, $\rho_e\mathbf{E}$ representing the '*Coulomb force*' as compared to the di-electrophoretic force term $(-\tfrac{1}{2}E^2\nabla\varepsilon)$ for most dielectric fluid in Ref. 60 *Hz AC* electric field. Moreover, we shall assume that the mass density ρ and the dielectric constant ε are linearly dependent on temperature field as

$$\rho=\rho_0[1-\alpha(T-T_0)],\quad \varepsilon=\varepsilon_0[1-e(T-T_0)], \tag{8}$$

where α is the coefficient of volume expansion and e is the coefficient of relative variations of the dielectric constant with temperature, which is assumed to be small. Modifying the pressure term using the equation

$$P=p-\frac{1}{2}\rho\frac{\partial\varepsilon}{\partial\rho}E^2. \tag{9}$$

Now, equation of motion (2) with the help of Equations (7) and (9) can be written as

$$\rho\left[\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v}\right]=-\nabla P+\rho\mathbf{g}+(\mu+\kappa)\nabla^2\mathbf{v}+\kappa\nabla\times\mathbf{\Pi}-\frac{1}{2}E^2\nabla\varepsilon, \tag{10}$$

after retaining only the di-electrophoretic force term.

We now define the basic state of the system as below

$$\begin{aligned}&\mathbf{v}=\mathbf{v}_b=0,\quad \mathbf{\Pi}=\mathbf{\Pi}_b=0,\quad P=P_b(z),\quad T=T_b(z)\\&=T_0-\beta z,\quad \varepsilon=\varepsilon_b(z)=\varepsilon_0(1+e\beta z),\quad \mathbf{E}=\mathbf{E}_b(z)\\&=\frac{E_0\widehat{k}}{1+e\beta z},\quad \phi_b(z)=-\frac{E_0}{e\beta}\log(1+e\beta z),\end{aligned} \tag{11}$$

where $\beta=(T_0-T_1)/d$, is the adverse temperature gradient, $E_0=-\phi_1 e\beta z/\log(1+e\beta z)$, is the root mean square value of the electric field at $z=0$, $\widehat{k}$ is the unit vector in the direction of positive z-axis and the subscript b denotes the quantity at basic state.

Let the initial state be slightly disturbed from the basic state, then all the parameters describing the system will undergo perturbation. We shall study the stability of the basic state by introducing the following perturbations

$$\begin{aligned}&\mathbf{v}=\mathbf{v}_b+\mathbf{v}',\quad \mathbf{\Pi}=\mathbf{\Pi}_b+\mathbf{\Pi}',\quad P=P_b+P',\quad \mathbf{E}=\mathbf{E}_b+\mathbf{E}',\\&T=T_b+T',\quad \rho=\rho_b+\rho',\quad \phi=\phi_b+\phi',\quad \varepsilon=\varepsilon_b+\varepsilon',\end{aligned} \tag{12}$$

where $\boldsymbol{v}'=(u,v,w)$, $\boldsymbol{\Pi}'$, P', T', $\boldsymbol{E}'$, ρ', ϕ' and ε' are perturbations from the base values in the corresponding quantities. Plugging (12) into Equations (3)–(6) and (10), owing to the linear stability theory