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On two-dimensional finite amplitude electro-convection in a dielectric liquid induced by a strong unipolar injection

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ABSTRACT

The hydrodynamic stability of a dielectric liquid subjected to strong unipolar injection is numerically investigated. We determined the linear criterion T_c (T being the electric Rayleigh number) and finite amplitude one T_f over a wide range of the mobility parameter M. A noticeable discrepancy is shown for T_f between our numerical prediction and the value predicted by stability analysis, which is due to the velocity field used in stability analysis. Recent studies revealed a transition of the flow structure from one cell to two with an increase in T. We demonstrate that this transition results in a new subcritical bifurcation.

Introduction

Electro-convection induced by the unipolar charge injection into an insulating liquid is a fundamental problem in Electro-Hydro-Dynamics (EHD) [1,2]. The electro-chemical reaction at the interface between liquid and electrode gives rise to injection of ions [3], and the Coulomb force acting upon these injected free charges tends to destabilize the system and induce the flow motion. This type of flow motion plays the center role in several industry applications, such as heat transfer enhancement [4,5] and flow control [6]. However, the inherent strong and complex nonlinear couplings in such a system make the problem difficult to analyze. For *homogeneous* and *autonomous* injection between two parallel planar electrodes, there are two basic features in the hydrodynamic stability. First, the hydrostatic state is potentially unstable. When the driving parameter exceeds a critical value, the instability sets in and flow motion takes place. Linear stability analysis shows that the linear criterion, which is a function of the electric Rayleigh number T, is highly dependent on the injection level C but independent on the dimensionless mobility parameter M [7]. Second, the linear bifurcation is subcritical and there exists a nonlinear instability criterion. This feature is due to the ion drift mechanism, which states that charge carriers migrate with a finite ionic velocity under the effect of electric field. The competition between the ionic velocity and the fluid velocity leads to the formation of the so-named charge void region [8]. Since the finite amplitude criterion is lower than the linear one, a hysteresis loop is established between them. The physical mechanism for the subcritical bifurcation was first deduced by Félici with a simplified hydraulic model of 2D rolls in the weak injection regime [9]. In that paper, the author proved that the maximum fluid velocity should be higher than the ionic velocity in order to sustain a stable electro-convective motion. The case of strong injection regime was later discussed by Atten and Lacroix [10]. Both the cellular patterns of 2D rolls and 3D hexagonal cells were considered. With some assumptions, such as the infinite M number and the number of modes retained for the approximation of the velocity field, the nonlinear criteria for various injection levels were determined. For $C = 10$, the nonlinear criteria for 2D rolls with one mode and two modes, 3D hexagonal cells with one mode were found to be about 125.0, 116.0 and 111.7, respectively [10]. These values have been widely compared with experimental and numerical results.

The subcritical bifurcation phenomenon has been qualitatively confirmed by experiments [11,12]. In Ref. [12], Atten and Lacroix reported the experimental results of the Space Charge Limited (SCL)

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regime ($C \to \infty$), and they illustrated the linear and finite amplitude criteria associated with the hysteresis loop with the current–voltage characteristics. Moreover, a hexagonal convective pattern was observed at the motion threshold [12,13], which is consistent with the theoretical prediction [10,13]. However, the theoretical predictions of the linear and finite amplitude stability criteria were 160.75 [7] and 110.0 [10], while experimental findings were about 110 and 90 respectively [12]. In addition, experimental observation revealed that the flow pattern at the motion threshold was not perfectly steady but always exhibited strong fluctuations around a well defined average. These discrepancies between theoretical analysis and experimental findings have not been well explained until now [14].

Since the numerical simulation can provide the dynamic of all basic fields, it is an efficient tool to gain additional insights into electro-convective phenomena. In simulating Coulomb-driven flows, the solving of charge density equation plays a central role [15,16]. The contribution of the molecular diffusion to charge transport is minimal [17], and thus it is often neglected. As a consequence the charge density equation is convection-dominated, and dedicated numerical algorithms are required to capture the steep gradient and simultaneously avoid unphysical oscillations in the charge density distribution [18,19]. To date, several numerical methods have been developed to model Coulomb-driven flows, see the review paper [15]. A numerical solver is desirable to accurately reproduce both the linear and finite amplitude criteria. However, early attempts failed with the finite amplitude one because of the serious numerical diffusion resulting from the low order discretization schemes or coarse grids [8,19]. In Ref. [20], Chicón et al. developed a particle-in-cell (PIC) method for the charge density equation. To simplify the problem, the velocity field in Ref. [20] was not obtained by solving Navier–Stokes equations. Instead it was computed with an analytical expression, which was derived based on the assumption that the flow takes a form of 2D self-similar roll. We call this strategy of simulation as the Imposed Velocity Field (IVF) approach [19]. This is to be compared with the strategy of solving Navier–Stokes equations (SNS approach). Based on the IVF approach, Chicón et al. [20] found 121.4 for the finite amplitude criterion with $C = 10$, which is close to 125, the value predicted by the nonlinear stability analysis. The case of $C = 10$ has been commonly considered in numerical studies to represent a strong injection regime, all results listed below are with this strength. In Ref. [21], the same IVF strategy was utilized and the charge density equation was solved by the PIC method and a flux corrected transport (FCT) algorithm. The good agreement between the results obtained with the two methods was highlighted. Recently, the complete set of governing equations for EHD convection was successfully solved by several groups. In Ref. [22], a well tested direct numerical simulation code [23] was modified to include the electrical equations. For 2D cases with $M = 60$, the obtained linear and finite amplitude criteria are about 162.0 and 110.0, respectively. In Ref. [24], Vázquez et al. extended their previous works by using a finite element (FEM) solver for Navier–Stokes equations. In Ref. [25], both 2D and 3D electro-convection were studied with the commercial software. However, only results of the linear stability criterion were reported in Refs. [24,25]. In Ref. [26], all equations were solved with a finite volume method (FVM), and a total variation diminishing (TVD) scheme [27] was applied to the charge equation. For $M = 10$, they found 155.64 and 107.5 for the linear and finite amplitude criteria, respectively. More recently, Vázquez and Castellanos developed a Discontinuous Galerkin FEM based algorithm [28]. For $M = 20$, they found 108.7 for the finite amplitude criterion. In another recent paper [18], a comparison between the results obtained by IVF and SNS was performed. For $M = 40$, the finite amplitude criteria obtained with IVF and SNS were about 121.0 and 109.0, respectively [18]. The most important results of the finite amplitude criterion are summarized in Table 1.

It is clear that previous numerical results covered a wide range of M. This is reasonable, since typical values of M for dielectric liquids vary in a wide range ($M \geq 3$) [2,17]. For example, the value of M for H^+ in ethanol is 4.1, while of Cl^- in chlorinated diphenyls in the range of [60, 475] [17]. In addition, the reported finite amplitude criteria are slightly different from one to the other. The difference may be due to numerical reasons (e.g. numerical diffusion and oscillations) since different numerical techniques were used. As will be demonstrated in this study, the relationship between the finite amplitude criterion and M also partially contributes to this difference. More importantly, we are surprised to see that all results with the SNS approach, comparing to the IVF approach, are even farther away from the analytical values predicted by the stability analysis. It is well proved that the linear criterion is independent on M [7]. However, concerning the finite amplitude criterion, there is not a detailed study on its dependence on M. Using an FD scheme, Castellanos and Atten [19] found different values of the finite amplitude criterion for $M = 60$ and $M \to \infty$ in the weak injection regime. However, their numerical approach was unable to resolve accurately the charge distribution, and their critical values were largely overestimated. On the other hand, Atten and Lacroix [10] called the attention on the possible dependence of the finite amplitude criterion on M in their conclusions. However, they did not address this issue in detail.

Numerical studies with strong injection have been extended to high values of T with the aim of determining the route to chaos or turbulence [24–26,28,29]. In Ref. [24], a transition of the flow structure from one convective cell to two cells with $T = 400$ and $M \approx 63.2$ was reported. Such a transition arises due to the nonlinear instability with high values of driving parameters, and it leads to a new bifurcation of the system. The same transition has been confirmed in later studies with different values of M [25,26,28,29]. In this paper we attempt to deepen the study by investigating the route that the two-cell flow structure returns to rest when T is decreased. The obtained results show a complete bifurcation diagram in the finite amplitude regime.

The main objective of this study is to determine the relationship between the finite amplitude criterion and M. The other goal is to extend the numerical bifurcation study to high electric Rayleigh numbers. The paper is organized as follows. In the next section the formulation of the physical problem is described. In Numerical methods section, the numerical methods are explained. In Results and discussions section, our numerical findings are reported and finally conclusions are given in the last section.

Table 1
Previous analytical and numerical predictions of the finite amplitude criterion for $C = 10$.

Authors	Strategy	M	Results
Atten and Lacroix [10]	Analytical, one-mode[a]	$+\infty$	125.0
	Analytical, two-mode[b]	$+\infty$	116.0
Chicón et al. [20]	IVF, PIC, one-mode[a]	[5, 40], $+\infty$	121.4
Cerizza [22]	SNS, 4th compact FD scheme	60	110.0
Traoré and Pérez [26]	SNS, FVM TVD scheme	10	107.5
Vázquez and Castellanos [28]	SNS, FEM Discontinuous Galerkin	20	108.7
Traoré and Jian [18]	IVF, FVM TVD scheme, one-mode[a]	40	121.0
	SNS, FVM TVD scheme		109.0

[a] and [b]: one-mode and two-mode mean the velocity formula used to approximate the two-dimensional rolls.