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A fractal streaming current model for charged microscale porous media

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ABSTRACT

An analytical expression for the streaming current in fractal porous media is developed based on the capillary model and the fractal theory for porous media. The proposed fractal model is expressed as a function of the space charge density at the solid–liquid interface, the fluid flow rate, the Debye–Huckel parameter, the minimum and maximum pore/capillary radii and fractal dimensions for porous media. The results are compared with available experimental data and good agreement is found between them. In addition, factors influencing the streaming current in porous media are also analyzed.

Introduction

It is well known that most solid surfaces of natural and artificial objects carry electrostatic charges, which will produce an electrical surface potential such as in insulative pipes and porous media. When a liquid with a very small number of ions flows through a pipe or a porous medium, there is the current in the fluid. Rice and Whitehead [1] derived an expression for the current (i) in a cylindrical capillary, i.e.

$$i(r) = \frac{\pi r^2 \rho_w}{\mu \kappa^2}\left\{\frac{\Delta P}{L_0}\left[1 - \frac{2I_1(\kappa r)}{\kappa r I_0(\kappa r)}\right] + E_0\rho_w\left[1 - \frac{2I_1(\kappa r)}{\kappa r I_0(\kappa r)} - \frac{I_1^2(\kappa r)}{I_0^2(\kappa r)}\right]\right\} - \pi r^2 E_0 \sigma \tag{1}$$

where r is the capillary radius, ρ_w is the space charge density at the solid–liquid interface, κ is the Debye–Huckel parameter, ΔP is the pressure drop, L_0 is the characteristic length or the length of the straight line/capillary along the macroscopic pressure gradient in the medium, E_0 is the applied electric field, μ and σ are the viscosity and conductivity of the fluid, I_0 and I_1 are the zero-order and first-order modified Bessel functions, respectively. It can be seen that on the right side of Eq. (1) the first term is the current due to transport of charge by the fluid, and the second term is the conduction current. If the capillary is tortuous and $E_0 = 0$, Eq. (1) is rewritten as

$$i(r) = \frac{\pi \Delta P \rho_w}{\mu L_\tau}\frac{r^2}{\kappa^2}\left[1 - \frac{2I_1(\kappa r)}{\kappa r I_0(\kappa r)}\right] \tag{2a}$$

In general, the radius r (its scale is micron) is far larger than the thickness δ_0 ($\delta_0 = \kappa^{-1}$~2nm for water used in present work) of the diffuse layer. Thus, $\kappa r >> 1$, and then the function $(1-2I_1(\kappa r)/\kappa r I_0(\kappa r))$ [1] from Eq. (2a) tends to 1. So Eq. (2a) can be simplified as

$$i(r) = \frac{\pi \Delta P \rho_w}{\mu L_\tau}\frac{r^2}{\kappa^2} \tag{2b}$$

with L_τ the tortuous/real length. L_τ follows the fractal scaling law given by Ref. [2]

$$L_\tau(r) = (2r)^{1-D_\tau} L_0^{D_\tau} \tag{3}$$

where D_τ is the fractal dimension for tortuous capillaries with $1 < D_\tau < 2$ in two dimensions and $1 < D_\tau < 3$ in three dimensions.

In Eq. (2), $i(r)$ is called the streaming current, which is an electrokinetic phenomenon in many fields such as soil physics [3–7], petroleum and electrical industries [8–10]. It has tremendously caught the attention of a large number of researchers [11–20]. Paillat et al. [13,15] performed a series of experiments and theoretical analysis on the streaming current in porous media with different

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hydraulic radii. Thus, the hydraulic radius model for the total streaming current through porous media can be expressed as [13],

$$I_h = \frac{8\rho_w Q_h}{(\kappa r_h)^2} \tag{4a}$$

where Q_h is the total flow rate through porous media, r_h is the hydraulic radius, and κ is the Debye–Huckel parameter and is given by Ref. [13]

$$\kappa = \sqrt{\frac{\sigma}{\varepsilon D_0}} \tag{4b}$$

with D_0 being the mean ion diffusion coefficient, and ε is the permittivity of the liquid. In addition, Moreau et al. [16] measured the streaming current in a glass pipe by a special equipment and found that the intensity of streaming current depends mainly on the space charge density at the wall and the electrical conductivity of liquid. Wu et al. [20] studied the electrokinetic flow and electric current in a fibrous porous medium constructed by an ordered array of circular cylinders. Moreover, many scholars [14,18,19] have theoretically investigated the streaming current in a cylindrical pipe. Their theoretical analysis depends on Rice and Whitehead's theory [1] and the hydraulic radius [21] for porous media. So far, the fractal geometry theory and technique were rarely applied to analyze the streaming current in porous media.

It has been shown that many natural porous media usually have extremely complicated and disordered pore structure with pore sizes extending over several orders of magnitude and their pore spaces have the statistical self-similarity and fractal characters. The fractal geometry [22,23] has successfully been employed to study the transport properties in porous media. Yu et al. [2,24] proposed a fractal model for permeability of porous media. Cai et al. [25,26] proposed analytical expressions for predicting imbibition rate and permeability of porous media. Xiao et al. [27,28] discussed the thermal conductivity of nanofluids with Brownian motion effect and permeabilities of fibrous gas diffusion layer in proton exchange membrane fuel cells based on the fractal geometry, respectively. A generalized modeling [29] of spontaneous imbibition was presented based on Hagen–Poiseuille flow in tortuous capillaries with variably shaped apertures. In addition, Zhu et al. [30] used the fractal theory to derive an analytical permeability of porous fibrous media with consideration of electrokinetic phenomena. The model is expressed as a function of porosity, dimensionless local averaging net charge density and dimensionless electric resistance number.

The purpose of this paper is to apply the capillary tube model and the fractal theory for porous media to derive an analytical model for the streaming current of viscous flow through porous media. In the next section, the fractal theory for porous media is introduced briefly.

Fractal theory for porous media

It is assumed that a porous medium is comprised of a bundle of tortuous capillaries with variable sizes. Furthermore, the cumulative size-distribution of pores in porous media has been proven to follow the fractal scaling law [2,31]:

$$N(\geq r) = (r_{\max}/r)^{D_f} \tag{5}$$

where N is the number of pores/capillaries, $r_{\max}$ is the maximum radius of capillary, D_f is the fractal dimension for pore space, $0 < D_f < 2$ in two-dimensional space and $0 < D_f < 3$ in three-dimensional space. Usually, there are numerous capillaries in porous media, Eq. (5) can be considered as continuous and differentiable function. Thus, differentiating Eq. (5) with respect to r yields

$$-\mathrm{d}N = D_f r_{\max}^{D_f} r^{-(D_f+1)} \mathrm{d}r \tag{6}$$

where $-\mathrm{d}N > 0$. Eq. (6) represents the number of pores from the radius r to the radius $r + \mathrm{d}r$. On the basis of Eq. (5), the total number of pores/capillaries from the minimum radius $r_{\min}$ to the maximum radius $r_{\max}$ can be obtained by,

$$N_t(\geq r_{\min}) = (r_{\max}/r_{\min})^{D_f} \tag{7}$$

Dividing Eq. (6) by Eq. (7) results in

$$-\frac{\mathrm{d}N}{N_t} = D_f r_{\min}^{D_f} r^{-(D_f+1)} \mathrm{d}r = f(r)\mathrm{d}r \tag{8a}$$

where

$$f(r) = D_f r_{\min}^{D_f} r^{-(D_f+1)} \tag{8b}$$

with $f(r)$ the probability density function. According to the probability theory, the function $f(r)$ should satisfy the following normalization relationship:

$$\int_{r_{\min}}^{r_{\max}} f(r)\mathrm{d}r = 1 - \left(\frac{r_{\min}}{r_{\max}}\right)^{D_f} = 1 \tag{9}$$

As a result, Eq. (9) holds if and only if [31],

$$(r_{\min}/r_{\max})^{D_f} = 0 \tag{10}$$

Eq. (10) can be regarded as a criterion whether the fractal theory and technique can be used to analyze the fractal characters of porous media. In general, $r_{\min}/r_{\max} \sim 10^{-2}$ or $<10^{-2}$ in porous media, and thus Eq. (10) holds approximately.

Eqs. (5)–(10) are the theoretical base of the present work.

Fractal model

The total flow rate

The flow rate through a single tortuous capillary is governed by Hagen–Poiseuille equation [32],

$$q(r) = \frac{\pi r^4 \Delta P}{8\mu L_\tau} \tag{11}$$

Based on Eqs. (3) and (6), the total flow rate Q through a unit cell in a porous medium can be obtained by integrating Eq. (11) from the minimum pore radius $r_{\min}$ to the maximum pore radius $r_{\max}$ as [2]:

$$Q = \int_{r_{\min}}^{r_{\max}} q(r)(-\mathrm{d}N) = \frac{\pi \Delta P}{2^{4-D_\tau}\mu} \frac{r_{\max}^{3+D_\tau}}{L_0^{D_\tau}} \frac{D_f}{3 - D_f + D_\tau}\left(1 - \alpha^{3-D_f+D_\tau}\right) \tag{12}$$

where $\alpha = r_{\min}/r_{\max}$. Since $0 < D_f < 2$ and $1 < D_\tau < 3$, $\alpha^{3-D_f+D_\tau} \approx 0$. So Eq. (12) is reduced to