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Electrohydrodynamic flow analysis in a circular cylindrical conduit using Least Square Method

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ABSTRACT

In this article, Electrohydrodynamic flow (EHD flow) in a circular cylindrical conduit is studied by a semi-exact and high efficient weighted residual method called Least Square Method (LSM). A principle of LSM is briefly introduced and later is employed to solve the described problem. Furthermore, the effects of the Hartmann electric number (Ha) and the strength of nonlinearity (α) on velocity profiles are discussed and presented graphically. Results are compared with numerical solution and obtained residuals are compared with those of HAM which previously were done by Mastroberardino in Ref. [3]. Outcomes reveal that LSM has an excellent agreement with numerical solution; also depicted residual functions showed that LSM is more acceptable than HAM especially for large values of Ha and α numbers, also it is simpler and needs fewer computations.

1. Introduction

The electrohydrodynamic flow (EHD flow) of a fluid in an "ion drag" configuration in a circular cylindrical conduit (see Fig. 1) is governed by a nonlinear second-order ordinary differential equation [1–3]:

$$\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}+Ha^2\left(1-\frac{w}{1-\alpha w}\right)=0,\quad 0<r<1. \tag{1}$$

subject to the boundary conditions

$$\left.\frac{dw}{dr}\right|_{r=0}=0,\quad w(1)=0. \tag{2}$$

where $w(r)$ is the fluid velocity, r is the radial distance from the center of the cylindrical conduit, Ha is the Hartmann electric number, and the parameter α is a measure of the strength of the nonlinearity. It has been noted that the nonlinearity confronted in this problem is in the form of a rational function, and thus, poses a significant challenge in regard to obtaining analytical solutions. Despite this fact, some analytical solutions are presented by researchers which following are introduced.

In 1997, McKee et al. [1] developed perturbation solutions in terms of the parameter α governing a nonlinear problem. McKee and his coworkers used a Gauss–Newton finite-difference solver combined with the continuation method and Runge–Kutta shooting method to provide numerical results for the fluid velocity over a large range of values of α. This was done for both large and small values of α. In 1997, Paullet [2] proved the existence and uniqueness of a solution of BVP of electrohydrodynamic flow and in addition, he claimed an error in the perturbation and numerical solutions given by McKee [1] for large values of α. Recently Mastroberardino [3] presented the approximate solution by homotopy analysis method [4–7] (HAM) for the nonlinear BVP governed by electrohydrodynamic flow of a fluid in a circular cylindrical conduit for $\alpha \in (0,1)$. He showed that HAM solutions are quite accurate especially for lower values of the parameters α and H^2, but the accuracy decreases rather fast for higher values of these parameters. Khan et al. [8] introduced new homotopy perturbation method [9–13] and Pandey et al. [14] presented two semi-analytic algorithms to solve this equation for various values of relevant parameters based on optimal homotopy asymptotic method (OHAM) and optimal homotopy analysis method.

There are some simple and accurate analytical techniques for solving differential equations called the Weighted Residuals Methods (WRMs). Collocation, Galerkin and Least Square are

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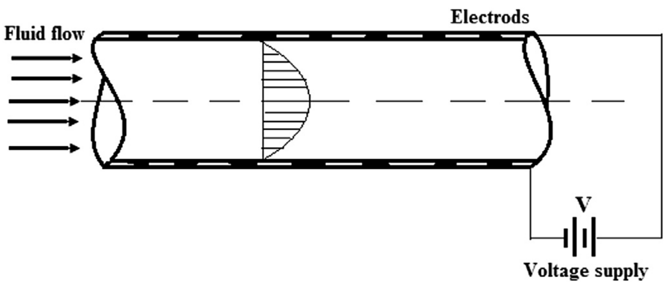

Fig. 1. Schematic of the EHD flow in a circular cylindrical conduit.

examples of the WRMs. Stern and Rasmussen [15] used collocation method for solving a third order linear differential equation. Vaferi et al. [16] have studied the feasibility of applying of Orthogonal Collocation method to solve diffusivity equation in the radial transient flow system. Hendi and Albugami [17] used Collocation and Galerkin methods for solving Fredholm–Volterra integral equation. Recently Least Square Method is introduced by Aziz and Bouaziz [18] and is applied for a predicting the performance of a longitudinal fin [19]. They found that least squares method is simple compared with other analytical methods. Shaoqin and Huoyuan [20] developed and analyzed least-squares approximations for the incompressible magneto-hydrodynamic equations.

According to the above explanations, we motivated to find a semi-exact analytical solution for electrohydrodynamic flow in which all α values (especially for $\alpha >> 1$) have a good agreement with numerical solution, so Least Square Method (LSM) is introduced. For large α values, LSM results are qualitatively similar with Paullet's solutions. Despite of simplicity of this method, it has lower residuals compared with HAM which presented in Ref. [3] for wide range of α and Ha numbers. Also, the effects of α and Ha numbers on velocity profile are discussed and treatment of the velocity profile near the centerline and walls is discussed in the present work.

2. Least Square Method (LSM)

There existed an approximation technique for solving differential equations called the Least Square Method (LSM). Suppose a differential operator D is acted on a function u to produce a function p:

$$D(u(x)) = p(x) \tag{3}$$

It is considered that u is approximated by a function $\tilde{u}$, which is a linear combination of basic functions chosen from a linearly independent set. That is,

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i \varphi_i \tag{4}$$

Now, when substituted into the differential operator, D, the result of the operations generally isn't $p(x)$. Hence an error or residual will exist:

$$R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{5}$$

The notion in LSM is to force the residual to zero in some average sense over the domain. That is:

$$\int_X R(x)\, W_i(x) = 0 \quad i = 1,2,\ldots,n \tag{6}$$

where the number of weight functions W_i is exactly equal the number of unknown constants c_i in $\tilde{u}$. The result is a set of n algebraic equations for the unknown constants c_i. If the continuous summation of all the squared residuals is minimized, the rationale behind the LSM's name can be seen. In other words, a minimum of

$$S = \int_X R(x)R(x)\mathrm{d}x = \int_X R^2(x)\mathrm{d}x \tag{7}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is,

$$\frac{\partial S}{\partial c_i} = 2\int_X R(x)\frac{\partial R}{\partial c_i}\mathrm{d}x = 0 \tag{8}$$

Comparing with Eq. (6), the weight functions are seen to be

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{9}$$

However, the "2" coefficient can be dropped, since it cancels out in the equation. Therefore the weight functions for the Least Squares Method are just the derivatives of the residual with respect to the unknown constants

$$W_i = \frac{\partial R}{\partial c_i} \tag{10}$$

3. Application of LSM on EHD flow analysis

Because trial function must satisfy the boundary conditions in Eq. (2), so it will be considered as,

$$w(r) = c_1\left(1-r^2\right) + c_2\left(1-r^3\right) + c_3\left(1-r^4\right) + c_4\left(1-r^5\right) \tag{11}$$

In this problem, according to the Eq. (5), residual function will be as,

$$\begin{aligned} R(r) = r\Big(& Ha^2 - 9c_2r - 16c_3r^2 - 25c_4r^3 + 4\alpha c_1^2 - Ha^2c_1 \\ & - Ha^2c_2 - Ha^2c_3 - Ha^2c_4 + 16c_3^2r^2\alpha + 4c_1\alpha c_3 + 4c_1\alpha c_2 \\ & - 4\alpha c_1^2r^2 - 16c_3^2r^6\alpha + 9c_2^2r\alpha - 9c_2^2r^4\alpha - Ha^2\alpha c_1 \\ & - Ha^2\alpha c_2 - Ha^2\alpha c_3 - Ha^2\alpha c_4 + Ha^2c_1r^2 + Ha^2c_2r^3 \\ & + Ha^2c_3r^4 + Ha^2c_4r^5 - 4c_1 + 25c_4^2r^3\alpha + 4c_1\alpha c_4 \\ & - 25c_4^2r^8\alpha - 13c_1\alpha c_2r^3 - 20c_1\alpha c_3r^4 - 29c_1\alpha c_4r^5 \\ & + 9c_2r\alpha c_1 + 9c_2r\alpha c_3 + 9c_2r\alpha c_4 - 25c_2r^5\alpha c_3 \\ & - 34c_2r^6\alpha c_4 + 16c_3r^2\alpha c_1 + 16c_3r^2\alpha c_2 + 16c_3r^2\alpha c_4 \\ & - 41c_3r^7\alpha c_4 + 25c_4r^3\alpha c_1 + 25c_4r^3\alpha c_2 + 25c_4r^3\alpha c_3 \\ & + Ha^2\alpha c_1r^2 + Ha^2\alpha c_2r^3 + Ha^2\alpha c_3r^4 + Ha^2\alpha c_4r^5\Big) \end{aligned} \tag{12}$$

Table 1
Coefficients (A, B, C, D and E) for Eq. (14) in different Ha and α numbers.

Coefficient	$Ha^2 = 0.5$		$Ha^2 = 4$	
	$\alpha = 0.5$	$\alpha = 4$	$\alpha = 0.5$	$\alpha = 4$
A	0.1137465	0.1087386	0.497918	0.212637
B	−0.109954	−0.1006484	−0.348699	−0.511836
C	0.0001223	−0.0005611	0.0375124	1.756696
D	−0.0040636	−0.0106654	−0.191238	−2.071364
E	0.0001491	0.0031363	0.004506	0.613867