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Analysis of the electrostatic force on a dielectric particle with partial charge distribution

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A B S T R A C T

This paper presents the analysis of the electrostatic force acting on a charged dielectric particle on a grounded plane. The force has been determined by a numerical field calculation method to make clear the effect of particle dielectric constant and charge distribution on the particle surface. The charge is treated to be distributed in three ways: (a) uniformly over entire surface, (b) partially on the upper, or (c) on the lower part of a particle. The calculation results show that, if a particle with dielectric constant $\varepsilon_p = 3$ is partially charged on the lower part by a zenith angle $\pi/2$, $\pi/4$ and $\pi/8$, the force shall be higher by 0.7, 4.3 and 20 times, respectively, than that for a uniform charging with the same charge amount. On the other hand, the force becomes weaker when charge is on the upper part. The effect of the particle dielectric constant is found to be dependent on the charge distribution. With charge uniform on the entire surface or on the upper part, the force always increases with the dielectric constant. However, when surface charge is restricted to a small area at the lower part of the particle ($\theta_q < \pi/4$), the force may decrease with increasing the dielectric constant.

1. Introduction

Charged dielectric particles are used in many applications of electrostatics such as powder coating and electrophotography. Electrostatic force acting on the particles in such applications plays an important role in their performance. For example, the detachment of charged toner particles from an electrode is done by applying electric field which must supersede the adhesive force between the particles and the electrode. A simplified configuration of an isolated, charged spherical particle above a grounded plane is commonly applied in the analysis of the force on the particle. The attractive force F_a between the particle and the plane is often estimated simply by the method of images as

$$F_a = \frac{Q^2}{16\pi\varepsilon_m\varepsilon_0(R+d)^2} \tag{1}$$

where R is the radius of the particle, d the height of the particle from the grounded plane, Q total charge on the particle, ε_m dielectric constant (relative permittivity) of the surrounding medium, and ε_0 the permittivity of free space. This equation is the force between a charge Q at the particle center at a height $h = R + d$ above the conducting plane and its image charge $-Q$ induced the plane. (Thus, the separation between the charges is equal to $2h$). It gives the exact solution only when a uniform surface charge density $\rho_s = Q/(4\pi R^2)$ is on the sphere surface and the effect of the particle dielectric constant ε_p, usually different from ε_m, is neglected. An accurate analysis of the electric and electrostatic force on a dielectric particle for various values of d was done by Davis [1] using bispherical coordinates and by Fowlkes and Robinson using multipole images [2]. These analytical works show that the effect of the dielectric constant of the particle is negligible only when the particle is far away enough from the plane (about 10 times of particle diameter). For a particle closer to the plane, the force is greatly enhanced by increasing the dielectric constant ε_p. For a constant ε_p, the force increases with decreasing d, but converges to an upper limit as d approaches zero. A real-number parameter α has been incorporated into Eq. (1) to include such effect of polarization on the adhesion (increasing F_a) for a range of ε_p, primarily in the case of uniform charging [3].

Measurement of the adhesive force on toners usually gives much larger values than that given by Eq. (1) [3]. A charge-patch model, which assumes nonuniform distribution of charge on the particle surface, has been proposed to explain the difference between the analytical and experimental results [3,4]. In this model, force is determined by simply assuming that the charge-patch and the

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grounded plane form a parallel-plate capacitor [3]. However, this model provides only a rough approximation which takes into account neither curved surfaces nor the dielectric constant of the particle. A few numerical computations have been performed by the Galerkin-type Finite Element Method (FEM) for nonuniform charge distributions. One of them expresses the surface charge density as a linear sum of Legendre functions [5], which makes it rather difficult to apply the result to practical charge distributions. Another paper considers only the specific case of dumb-bell type charge distribution [6], in which a particle is charged in the angle range of 10° on the both sides of the particle.

In this paper, we study the behavior of the adhesive electrostatic force between a charged dielectric particle and a ground plane when the particle is uniformly charged on a portion of its surface. We focus on more fundamental configurations, in which the surface charge is either on the lower part or on the upper part of the particle. Instead of using the simple capacitor model, we apply the boundary element method (BEM) to calculate the electric field on the particle surface. With the BEM, which is a numerical method, we can obtain the field solution for various conditions of charge distribution and particle dielectric constant. Furthermore, the BEM usually enables more accurate calculation than the FEM for such arrangements with very narrow regions between a particle and an electrode where electric field is often concentrated. The force is then determined from the Maxwell stress due to the electric field on the particle surface. The objective of this study is to clarify the effect of the partial charge distribution on the particle as well as the role of the particle dielectric constant on the attractive force. Note that we confine our interest here to the electrostatic force based on continuous representation of charge by the surface charge density. Other discussion on non-electrostatic force and the other studies on toner adhesion may be referred to [7] and the references therein. The idea of proximity force has also been presented in several papers to explain the difference between the theoretical prediction and the experimental results [8].

2. Configuration of analysis and parameters

We consider the configuration of a spherical, dielectric particle located on a grounded plane as shown in Fig. 1. The particle, having a radius R, is either (a) charged uniformly over its surface, (b) partially charged at the top or (c) partially at the bottom. These charging conditions are schematically shown in Fig. 2. For the case of the partially charged particle, θ_q denotes the boundary angle of the charged area. In all cases, the surface charge density is treated to be constant ($=\rho_s$) on the charged area. The configuration of charge on the lower part represents cases in which a large amount of charge accumulates on a small portion of the particle surface. By the electric force, this charged region tended to be attracted so as to be in contact with the plane. On the other hand, the charging model in Fig. 2(b) is used to study the variation of force if the charged region is far from the contact point and the plane. The dielectric constant of the particle is denoted by ε_p, and that of the surrounding medium is assumed to be unity (free space) for simplicity.

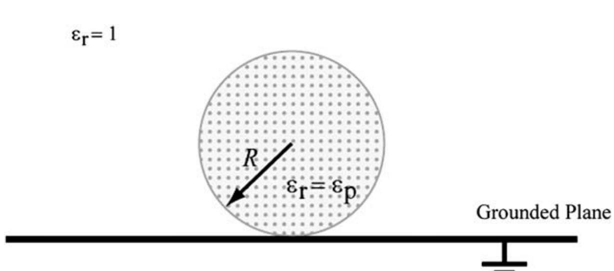

Fig. 1. Configuration of a dielectric particle resting on a grounded plane.

On the surface of the particle, the boundary conditions are as follows:

1. Continuity of potential.

$$\phi^{ext} = \phi^{int}, \tag{2}$$

where ϕ is the potential and the superscripts 'ext' and 'int' are used to indicate that the value is on the exterior or the interior side of the surface, respectively.

2. Gauss's law.

With the normal component E_n of the electric field on the particle surface,

$$\begin{aligned} E_n^{ext} - \varepsilon_p E_n^{int} &= \frac{\rho_s}{\varepsilon_0} \quad \text{on the charged area, and} \\ &= 0 \quad \text{on the uncharged area} \end{aligned} \tag{3}$$

where ρ_s is the surface charge density. In this equation, E_n^{ext} and E_n^{int} are considered to be positive in the direction from the interior to the exterior of the particle.

When the particle partially charged on the upper part approaches the grounded plane, its position may become unstable and not exist in practice if the particle shape is perfectly spherical. This is because if the particle is slightly rotated in the θ direction from the position in Fig. 2(b), the net electrostatic force will have both vertical and horizontal components and attract the charged portion to the conducting plane. However, it is still worth seeing the variation of the force with charge position. In the numerical analysis, we use the following values of θ_q and ε_p:

$\theta_q = \pi/2$, $\pi/3$, $\pi/4$, $\pi/6$, and $\pi/8$,
$\varepsilon_p = 1$, 3 and 5 (in some cases, also 2 or 4).

This range of ε_p includes the dielectric constant of toner particles, which is usually between 3 and 4.

3. Calculation method

The boundary element method (BEM) for axisymmetrical (AS) arrangements is used for the calculation [9]. Owing to the axisymmerty of the configuration, the boundary of the particle can be represented by the contour $\theta = 0$–π ($r = R$), which is subdivided into line elements in the BEM. At any point x on each element, we express the potential $\phi(x)$ as

$$\phi(x) = \sum_i \phi_i N_i^{\phi}(x) \tag{4}$$

and the outward normal component of the electric field $E_n(x)$ as

$$E_n(x) = \sum_i E_{n,i} N_i^{E}(x) \tag{5}$$

where N_i^{ϕ} and N_i^{E} are dimensionless functions of the position x on the element, which are used to interpolating ϕ and E_n from the values ϕ_i and $E_{n,i}$ at node i, respectively. The values of ϕ_i and $E_{n,i}$ are unknown and to be determined by a system of linear equations: