Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Electrostatics 64 (2006) 690–698
Journal of ELECTROSTATICS
www.elsevier.com/locate/elstat

Temporal linear instability analysis of an electrified coaxial jet with inner driving liquid inside a coaxial electrode

Fang Li, Xie-Yuan Yin, Xie-Zhen Yin*

Department of Modern Mechanics, University of Science and Technology of China Hefei, Anhui, 230027, PR China

Received 2 January 2005; received in revised form 13 December 2005; accepted 8 January 2006
Available online 3 February 2006

Abstract

In this paper, a temporal linear stability analysis is performed of a coaxial jet composed of two immiscible liquids inside a coaxial electrode. This analysis is carried out to investigate the case of an inner driving coaxial electrospray system. The assumption is made that the inner liquid has high electric conductivity, and the outer liquid is an insulating dielectric. The dimensionless dispersion equation for both the axisymmetric and non-axisymmetric modes is derived and solved numerically for the axisymmetric case. The effects of the relevant dimensionless parameters on the instability of the jet are discussed in detail. These parameters include the dimensionless electrostatic force E, the dielectric constant ratio ε, the diameter ratios a and b, the velocity ratio Λ, the density ratio S, the Weber number, and the interface tension ratio ζ. Two independent unstable modes, *modes 1* and *2*, are found and analyzed. Among the various parameters, the dimensionless electrostatic force and the dielectric constant have a similar and remarkable influence on *modes 1* and *2*, altering drastically the regime of the jet as they vary. The interface tension on the outer interface promotes the instability of both *modes 1* and *2* in the region of long wavelengths while suppressing the growth rate in the region characterized by short wavelengths. The interface tension on the inner interface, however, promotes instability of only *mode 2* in the same way. The diameter ratio a has a great effect on *mode 2* while a negligible influence on *mode 1*. And the diameter ratio b has a slight effect on both the unstable modes.

Keywords: Electrospray; Coaxial jet; Dispersion equation; Temporal Instability

1. Introduction

Coaxial electrospray is a new, effective method of forming micro and nano capsules. It can be used in the drug industry, for injecting food additives, in paper manufacturing, and other industries as well. Recently, much experimental research has been performed to find an electrospray method for generating compound droplets [1]. Other research has focused on finding appropriate scaling laws between electric current and drop size for the cases of both outer driving and inner driving [2], as well as the different modes of the coaxial jet electrospray obtained for the outer driving case [3]. Until now, however, there has been little research dealing with theoretical and numerical modeling of this phenomenon. Particularly lacking has been any instability analyses of electrospray in the coaxial case.

As to the stability analysis of single-liquid electrospray, various results have been reported, including the effects of a wide range of parameters on the instability of the jet [4], the temporal linear stability of conductive and dielectric jets in both radial and axial electric fields [5], [6], the temporal linear stability analysis of a cylindrical electrified jet flowing at high velocity inside a coaxial electrode [7], the absolute and convective instabilities of a cylindrical electrified jet in a radial electric field [8], a nonlinear electro-hydrodynamic stability of a finite conducting jet in an axial electric field [9], and the stability of conducting viscosity jets in radial ac electric fields [10].

In a previous paper [11], we investigated the linear stability of an electrified coaxial jet with the outer liquid driven. In this paper, we attempt to explain the breakup of the electrified coaxial jet for the case where the inner liquid is driven. Specifically we utilize the method of temporal

*Corresponding author. Tel.: +86 551 3607645; fax: +86 551 3606459.
E-mail address: xzyin@ustc.edu.cn (X.Z. Yin).

0304-3886/$ - see front matter
doi:10.1016/j.elstat.2006.01.003

linear instability analysis. For the purpose of analysis, the two liquids and the ambient gas are assumed to be incompressible Newtonian fluids. The viscosities of all fluids are neglected, and the motion is assumed to be irrotational. The effects of gravity and magnetic fields are also neglected. The outer liquid and the gaseous phase are assumed to be perfect dielectrics; while inner liquid is conducting and obeys Ohm's law. Both conductivity and dielectric properties of all liquids are assumed constant over time. The dielectric relaxation time of the system is assumed so small that all free charges distribute on the liquid–liquid interface, achieving an equilibrium state, essentially instantaneously for both unperturbed and perturbed cases.

2. Theoretical Analysis

The coaxial jet considered in this paper consists of a cylindrical inner liquid jet of radius R_1, velocity U_1, and density ρ, and a coflowing outer liquid jet of radius R_2, velocity U_2, and density ρ_2. The background gas in the unperturbed case is stationary. An electric potential V_0, applied between the central axis anode and the earthed outer cylindrical cathode of radius R_3, is kept constant (Fig. 1). Hereafter in this paper, the subscripts 1, 2 and 3 shall denote the inner liquid, outer liquid, and background gaseous phase, respectively, when they are used to describe bulk physical quantities. These same subscripts will denote the inner liquid–liquid interface, the outer gas–liquid interface, and the cylindrical electrode, respectively, when used to describe interface or boundary physical quantities.

The viscosities of fluids are not considered, hence all shear forces at the liquid–liquid and gas-liquid interfaces disappear from the equations, and the basic flow velocities are allowed to sustain discontinuities at the various interfaces. In cylindrical coordinates (r,θ,z) , the basic velocity profiles are assumed to be

$$\vec{U}_1(r,\theta,z) = U_1(0,0,1),$$

$$\vec{U}_2(r,\theta,z) = U_2(0,0,1),$$

$$\vec{U}_3(r,\theta,z) = 0.$$

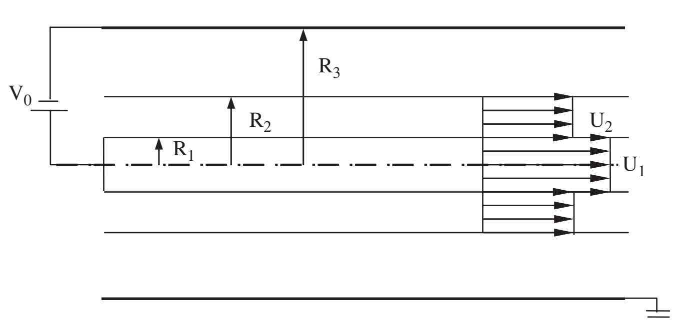

Fig. 1. Diagram of the model showing relevant coordinates and dimensions.

Because the entire inner jet of conducting liquid is equipotential, the potential and the electric-field intensity of the inner liquid become

$$V_1(r,\theta,z) = V_0, \quad \vec{E}_1 = -\nabla V_1 = 0.$$

The outer annular liquid jet and the background gaseous phase are both considered as dielectrics, and hence the electric fields inside them cannot be neglected. The potential and electric field intensity in the unperturbed case can be obtained via the usual electrostatic laws:

$$V_2 = V_0\left(\frac{\ln(r/R_1)}{\ln A} + 1\right), \ \vec{E}_2 = -\frac{V_0\vec{r}_0}{r\ln A}, \ R_1 \le r \le R_2,$$

$$V_3 = \frac{\varepsilon_2}{\varepsilon_3}\cdot\frac{V_0\ln(r/R_3)}{\ln A}, \ \vec{E}_3 = -\frac{\varepsilon_2}{\varepsilon_3}\cdot\frac{V_0\vec{r}_0}{r\ln A}, \ R_2 \le r \le R_3,$$

where $\vec{r}_0$ is the unit vector of the r- direction, and $A = (R_2/R_3)^{\varepsilon_2/\varepsilon_3}/(R_2/R_1)$.

During the process of linear stability analysis, we shall maintain the small amplitude disturbance assumption throughout. The interfaces being perturbed consist of the following:

$$r_{si} = R_i + \eta_i, \quad i = 1,2,$$

where η_i is the displacement of the interface from the unperturbed case.

The perturbed pressure field can be expressed as $p = p_0 + p'$. When the electric field is perturbed, it will still be assumed that $V_1(r,\theta,z) = V_0$, and $\vec{E}_1 = -\nabla V_1 = 0$. The potential and the electric-field intensity in the outer liquid and in the gaseous phase will be written as $V_i = V_{0i} + V'_i$, $\vec{E}_i = \vec{E}_{0i} + \vec{E}'_{ii}$, where subscript 0 denotes the unperturbed properties, and the 'prime' superscript denotes the perturbation of the corresponding quantity. In the normal mode method of temporal linear instability analysis, the wave-number k is real, and the frequency ω is complex function of k: $\omega(k) = \omega_r(k) + \mathrm{i}\omega_i(k)$. Hence the perturbation can be decomposed into the form of a Fourier exponential:

$$(\eta_i, \vec{u}'_i, p'_i, V'_i) = (\hat{\eta}_i(r), \hat{\vec{u}}_i(r), \hat{p}_i(r), \hat{V}_i(r))\mathrm{e}^{\omega t + \mathrm{i}(kz+n\theta)}, \tag{1}$$

where $\hat{\eta}_i$,$\hat{\vec{u}}_i$,$\hat{p}_i$,$\hat{V}_i$ are the perturbation amplitudes of the interface, velocity, pressure and electrical potential, respectively, and n is the azimuthal wave number.

Substituting (1) into the linearized, small perturbation equations for an inviscid fluid, we obtain a modified Bessel equation of order n for the amplitude function $\hat{p}_i(r)$:

$$\frac{\mathrm{d}^2\hat{p}_i}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\hat{p}_i}{\mathrm{d}r} - \left(k^2 + \frac{n^2}{r^2}\right)\hat{p}_i = 0. \tag{2}$$

At the same time, from Maxwell's equations applied to electro-hydrodynamics, we can obtain a modified Bessel equation of order n for the electrical potential perturbation amplitude $\hat{V}_i(r)$:

$$\frac{\mathrm{d}^2\hat{V}_i}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\hat{V}_i}{\mathrm{d}r} - \left(k^2 + \frac{n^2}{r^2}\right)\hat{V}_i = 0. \tag{3}$$