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Electrostatic field penetration through a slot on a conducting half-plane

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ABSTRACT

In this study, electrostatic field penetration through a slot on a conducting half-plane is investigated based on the Mellin transform and mode matching. To formulate the field behavior by the slotted half-plane, a slotted conducting wedge with a line charge is considered as an analysis model. A fast convergent series solution is obtained through the application of eigenfunction expansion and residue calculus. Numerical computations of the electrostatic field are conducted to demonstrate the penetration characteristics for the slotted conducting half-plane in terms of the slot dimensions and the position of the line charge. The computed results are also compared with simulated results.

1. Introduction

Electromagnetic wave scattering by a conducting half-plane is a canonical problem which has been the subject of many previous studies in diffraction theory [1–4]. Recently, the scattering problem of a half-plane with slots has attracted much attention, as this geometry is encountered in practical applications of antenna design and radar cross-sections [5,6]. The TM and TE diffraction solutions of a slotted conducting half-plane have been obtained by combining the Nystrom and Galerkin methods with field equivalence principles [5]. Scattering from a multiply slotted half-plane has been investigated based on the Kontorovich–Lebedev transform and by a mode-matching method [6]. Although dynamic field solutions for various structures of the half-plane have been presented, a static solution serves as a fundamental component of any dynamic field. In one study [7], an analysis of the electrostatic field by a charged half-plane was conducted, demonstrating that the derived static solution can be assumed as a spatial-harmonic function in the electromagnetic scattering problem.

The purpose of the present research is to investigate the electrostatic field from a line charge adjacent to a conducting half-plane with a slot. To complete the mathematical formulations for the slotted half-plane, a boundary-value problem of a slotted conducting wedge with a line charge is studied using the Mellin transform and a mode-matching method. Study for the electrostatic field of the slotted conducting half-plane is clearly an extension of earlier work [8] in which the electrostatic field from a line charge in a slotted hollow wedge was calculated with the Mellin transform. Numerical computations of the field behavior for the slotted conducting wedge and half-plane are performed to demonstrate the validity of the proposed method.

2. Field analysis

Fig. 1(a) shows the geometry of the slotted conducting half-plane with a line charge. The width of the slot on the half-plane is b-a, and the line charge ρ_ℓ, which is parallel to the z-axis, is located at $\rho = \rho'$ and $\phi = \phi'$. In the analysis that follows, the conducting planes are assumed to have a zero thickness and infinite conductivity. In order to utilize the Mellin transform and mode-matching method to analyze this slotted conducting half-plane, we consider the boundary-value problem of a slotted conducting wedge, as illustrated in Fig. 1(b). Here, the slot angle across the wedge is defined as α. The total electrostatic potential field in region (I) ($\alpha < \phi < 2\pi$) is represented as the sum of the primary and scattered components [8]:

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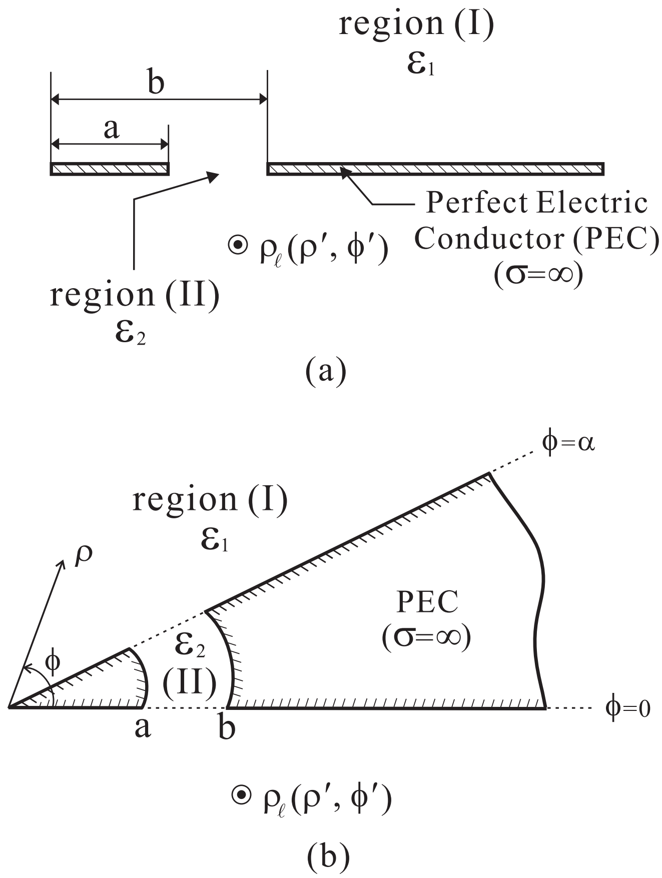

Fig. 1. (a) Slotted conducting half-plane with a line charge and (b) analysis model for applying the Mellin transform and mode-matching method.

$$\psi^I(\rho,\phi) = \psi_p^I(\rho,\phi) + \psi_s^I(\rho,\phi). \tag{1}$$

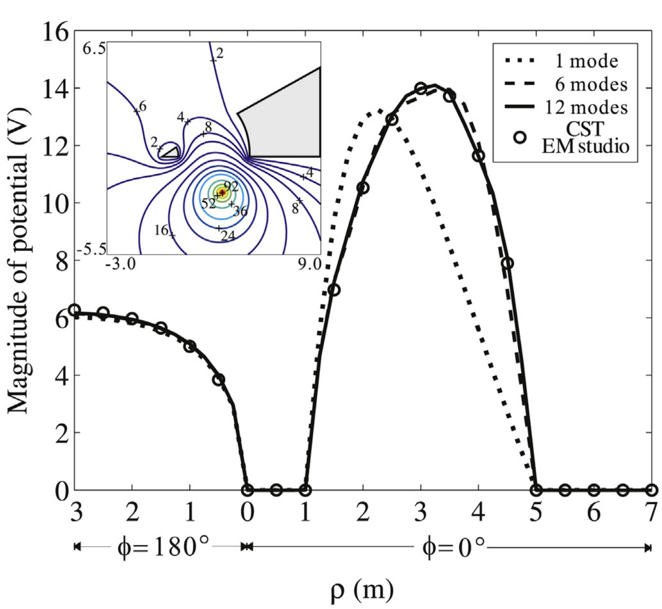

Fig. 2. Magnitude of the potential field on an aperture of a slotted conducting wedge with different numbers of modes when $a = 1.0$ m, $b = 5.0$ m, $\alpha = 30°$, $\varepsilon_{1r} = \varepsilon_{2r} = 1.0$, $\rho_\ell = 10^{-9}$ C/m, $\rho' = 4.0$ m, and $\phi' = 330°$.

The primary potential field due to the line charge ρ_ℓ in the vicinity of the slotted conducting wedge is

$$\psi_p^I(\rho,\phi) = -\frac{\rho_\ell}{\varepsilon_1}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left(\frac{\rho}{\rho'}\right)^{-\zeta}\frac{1}{\zeta\sin(\zeta(2\pi-\alpha))}\cdot \begin{cases}\sin(\zeta(\phi'-2\pi))\sin(\zeta(\phi-\alpha))d\zeta & \text{for } \alpha<\phi<\phi' \\ \sin(\zeta(\phi'-\alpha))\sin(\zeta(\phi-2\pi))d\zeta & \text{for } \phi'<\phi<2\pi\end{cases}. \tag{2}$$

Based on the Mellin transform [9], the scattered potential field in region (I) is given by

$$\psi_s^I(\rho,\phi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\rho^{-\zeta}\left[\tilde{\Phi}_1(\zeta)\cos(\zeta\phi) + \tilde{\Phi}_2(\zeta)\sin(\zeta\phi)\right]d\zeta. \tag{3}$$

The potential distribution in the slot (region (II): $a < \rho < b$ and $0 < \phi < \alpha$) is expressed by a series expansion because region (II) is a closed region bounded by two perfect electric conductors (PECs):

$$\psi^{II}(\rho,\phi) = \sum_{m=1}^{\infty}\sin(\phi_m \ln(\rho/b))[A_m\cosh(\phi_m\phi) + B_m\sinh(\phi_m\phi)], \tag{4}$$

where $\phi_m = m\pi/ln(a/b)$.

In order to determine the unknown modal coefficients A_m and B_m, field continuities are applied at each boundary. First, we employ Dirichlet boundary conditions at $\phi = \alpha$ and $\phi = 2\pi$, as

$$\psi^I(\rho,\alpha) = \begin{cases}\psi^{II}(\rho,\alpha) & \text{for } a<\rho<b \\ 0 & \text{otherwise}\end{cases} \tag{5}$$

and

$$\psi^I(\rho,2\pi) = \begin{cases}\psi^{II}(\rho,0) & \text{for } a<\rho<b \\ 0 & \text{otherwise}\end{cases}. \tag{6}$$

Applying the Mellin transform ($\int_0^\infty \rho^{\zeta-1}d\rho$) to Eqs. (5) and (6) yields two spectral coefficients:

$$\tilde{\Phi}_1(\zeta) = -\sum_{m=1}^{\infty}\frac{\phi_m F_m(\zeta)}{\sin(\zeta(2\pi-\alpha))}[A_m(\cosh(\phi_m\alpha)\sin(\zeta 2\pi) - \sin(\zeta\alpha)) + B_m\sinh(\phi_m\alpha)\sin(\zeta 2\pi)] \tag{7}$$

and

$$\tilde{\Phi}_2(\zeta) = \sum_{m=1}^{\infty}\frac{\phi_m F_m(\zeta)}{\sin(\zeta(2\pi-\alpha))}[A_m(\cosh(\phi_m\alpha)\cos(\zeta 2\pi) - \cos(\zeta\alpha)) + B_m\sinh(\phi_m\alpha)\cos(\zeta 2\pi)], \tag{8}$$

where $F_m(\zeta) = (b^\zeta - a^\zeta(-1)^m)/(\zeta^2 + \phi_m^2)$. Next, we employ the Neumann boundary conditions of

$$\varepsilon_1\frac{\partial\psi^I}{\partial\phi}\bigg|_{\phi=\alpha} = \varepsilon_2\frac{\partial\psi^{II}}{\partial\phi}\bigg|_{\phi=\alpha} \quad \text{for } a<\rho<b \tag{9}$$

and

$$\varepsilon_1\frac{\partial\psi^I}{\partial\phi}\bigg|_{\phi=2\pi} = \varepsilon_2\frac{\partial\psi^{II}}{\partial\phi}\bigg|_{\phi=0} \quad \text{for } a<\rho<b. \tag{10}$$

Multiplying Eqs. (9) and (10) by $\rho^{-1}sin(\phi_p ln(\rho/b))$ and integrating