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Transient lightning responses of grounding systems buried in horizontal multilayered earth with a hybrid method

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ABSTRACT

Combined with the Fast Fourier Transform (FFT), a mathematical hybrid method for accurately computing the lightning response from grounding systems buried in multilayered earth model has been developed in this paper. In the method, electrical circuit consists of "T" typical of basic elements. To accelerate calculations of the method, quasi-static complex image method and closed form of Green's function and analytical formula for mutual induction and impedance coefficients were introduced into this method. With the inverse FFT, the method can be used for studying performances of transient lightning response from the grounding systems.

1. Introduction

Recently, the computerized analysis about the hybrid method basing on quasi-static electromagnetic field theory has been developed [1–10]. The hybrid method can be used to study frequency behavior and lightning response of grounding systems.

The hybrid method has been developed out of conventional nodal analysis method [11]. The papers [1–6] were developed to analyze the behavior of grounding systems in frequency domain. And the methods in the papers [8,9] were used to study the lightning response of grounding systems. Among these papers, the papers [1–3] discussed the behavior of grounding systems in the frequency domain buried in only half infinite homogenous earth. Although Refs. [12,13] has developed the hybrid method basing on high frequency electromagnetic field theory, only half infinite homogenous earth model is considered. These confines in the frequency domain promoted a novel mathematical model to be developed in [4–7], with quasi-static complex image method (QSCIM) introduced, which can be used to fast calculate the currents distribution in the grounding systems buried in horizontal multilayered earth model in the frequency domain. The hybrid method can be combined with FFT, and the transient response from the grounding systems can be achieved. The lightning response of the grounding systems buried in infinite half homogenous earth has been developed in [8], and the lightning response of the grounding systems buried in horizontal multilayered earth has been discussed in [9].

Basic elements of the electrical circuit used by the hybrid method are "π" typical of basic elements (see (a) in Fig. 1) for all these papers [1–9]. However, the typical of basic elements can be further displaced with "T" typical of basic elements (see (b) in Fig. 1), which own better physical meanings. The hybrid method with "T" typical of basic elements has been developed in [10]. The hybrid method is basing on direct current electrical field theory and the behavior of the grounding systems buried in half infinite homogenous earth. The hybrid method can be further developed basing on quasi-static electromagnetic field theory to study the behavior of the grounding systems buried in horizontal multilayered earth in the frequency domain, and analyze the lightning response of the grounding systems buried in horizontal multilayered earth through the inverse FFT. It should be pointed out that the method is similar with the PEEC method, however, the PEEC method is developed to calculate the transient response to microwave integrated circuits in air space; Recently, the PEEC method has been developed to calculate the transient lightning response to the grounding systems [14], however, only uniform earth model has been considered.

In this paper, basing on previous work in [1–10], combined with the FFT, an accurate mathematical hybrid method with "T" typical of basic elements is developed for calculating the harmonic wave

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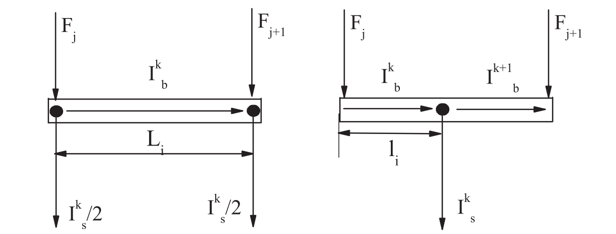

(a) "π" typical elementary element (b) "T" typical elementary element

Fig. 1. "T" typical of basic element.

currents from lightning currents distribution along the grounding systems buried in multilayered earth model. The earth is considered as horizontal multilayered earth model. To accelerate the calculation, QSCIM and closed form of Green's function were introduced, and the mutual inductive and conductive coefficients have analytical formulas so as to avoid the numerical integral.

2. Mathematical model of equivalent circuits with "T" typical of basic elements of the grounding systems in frequency domain

The transient problem is first solved by a formulation in the frequency domain. The time-domain response is then obtained by applying a suitable Fourier inversion technique.

First, a grounding system is divided in N_b pieces of segments that can be studied as an elemental unit, where the discrete grounding system has N_p end nodes and N_b middle nodes.

The grounding network is energized by injection of single frequency currents at one or more nodes. In general, we assume that a sinusoidal current source of value $\overline{F_i}$ is connected at ith node. Scalar electric potential (SEP) $\overline{V_j}$ of jth (i,j = 1,2,…,N_p) node on the grounding network referring to the infinite remote earth as zero SEP is defined. Meanwhile, we define a branch current $\overline{I_b^k}$, a branch voltage $\overline{U_b^k}$ and a leakage current $\overline{I_s^k}$ on kth (k = 1,2,…,N_b) segment.

With the above considerations and "T" typical of basic elements (see (b) in Fig. 1) is used, the obtained electric circuits may be studied using the conventional nodal analysis method [11]. For interactions of mutual induction among these discrete conductors, $2N_b$ pieces of conductors must be considered, which means each conductor has been separated into two parts due to its middle point. Then we have

$$[\bar{\mathbf{I}}_b] = [\bar{\bar{\mathbf{Y}}}_b][\bar{\mathbf{U}}_d] \tag{1}$$

where $[\bar{\bar{\mathbf{Y}}}_b]$ is a $2N_b \times 2N_b$ admittance matrix who includes mutual induction and internal impedance of conductors; $[\bar{\mathbf{U}}_d]$ is $2N_b \times 1$ vector of potential difference between end and middle points of conductors; $[\bar{\mathbf{I}}_b]$ is $2N_b \times 1$ vector of axis branch currents of conductors.

Considering an incidence matrix $[\bar{\bar{\mathbf{A}}}]$, the incidence matrix is $(N_b + N_p) \times 2N_b$ matrix and used to relate to branches and nodes (including middle and end points), whose elements can be referred in [4–7]. Then we know $[\bar{\bar{\mathbf{A}}}][\bar{\mathbf{I}}_b] = 0$ and $[\bar{\mathbf{U}}_d] = [\bar{\bar{\mathbf{A}}}]^t[\bar{\mathbf{V}}_n]$, here $[\bar{\mathbf{V}}_n]$ is $(N_b + N_p) \times 1$ vector of all SEP of nodes including end and middle points. Meanwhile, we know $[\bar{\mathbf{V}}_n] = [\bar{\mathbf{V}}_p/\bar{\mathbf{V}}_b]$ and $[\bar{\bar{\mathbf{A}}}] = \begin{bmatrix} \bar{\bar{\mathbf{A}}}_{\mathrm{pp}} & \bar{\bar{\mathbf{A}}}_{\mathrm{pb}} \\ \bar{\bar{\mathbf{A}}}_{\mathrm{pb}}^t & \bar{\bar{\mathbf{A}}}_{\mathrm{bb}} \end{bmatrix}$ and $[\bar{\bar{\mathbf{Y}}}_b] = \begin{bmatrix} \bar{\bar{\mathbf{Y}}}_{\mathrm{bb}} & \bar{\bar{\mathbf{Y}}}_{\mathrm{bb}} \\ \bar{\bar{\mathbf{Y}}}_{\mathrm{bb}} & \bar{\bar{\mathbf{Y}}}_{\mathrm{bb}} \end{bmatrix}$

So we have

$$\begin{bmatrix} \bar{\mathbf{F}} \\ -\bar{\mathbf{I}}_s \end{bmatrix} = \begin{bmatrix} \bar{\bar{\mathbf{A}}}_{pb}\bar{\bar{\mathbf{Y}}}_{bb}\bar{\bar{\mathbf{A}}}_{bp}^t & \bar{\bar{\mathbf{A}}}_{pb}\bar{\bar{\mathbf{Y}}}_{bb}\bar{\bar{\mathbf{A}}}_{bb}^t \\ \bar{\bar{\mathbf{A}}}_{bb}\bar{\bar{\mathbf{Y}}}_{bb}\bar{\bar{\mathbf{A}}}_{bp}^t & \bar{\bar{\mathbf{A}}}_{bb}\bar{\bar{\mathbf{Y}}}_{bb}\bar{\bar{\mathbf{A}}}_{bb}^t \end{bmatrix} \begin{bmatrix} \bar{\mathbf{V}}_p \\ \bar{\mathbf{V}}_b \end{bmatrix} \tag{2}$$

where $[\bar{\mathbf{F}}]$ is a $N_p \times 1$ vector of external currents sources; $[\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}]$ is the $N_b \times N_b$ branch admittance sub-matrix of the matrix $[\bar{\bar{\mathbf{Y}}}_b]$; $[\bar{\mathbf{I}}_s]$ is a $N_b \times 1$ vector of leakage currents; $\bar{\mathbf{V}}_p$ is a $N_p \times 1$ vector of SEP of the end points; $\bar{\mathbf{V}}_b$ is a $N_b \times 1$ vector of SEP of the middle points; The sub-matrices $[\bar{\bar{\mathbf{A}}}_{\mathrm{pp}}]$, $[\bar{\bar{\mathbf{A}}}_{\mathrm{pb}}]$ and $[\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}]$ are, respectively, $N_p \times N_p$, $N_p \times N_b$ and $N_b \times N_b$ matrices.

For interactions of mutual impedance among these discrete conductors, only N_b pieces of conductors should be considered, which means each conductor do not require to be separated into two parts due to its middle point, so we have

$$[\bar{\mathbf{V}}_b] = [\bar{\bar{\mathbf{Z}}}_s][\bar{\mathbf{I}}_s] \tag{3}$$

where $[\bar{\bar{\mathbf{Z}}}_s]$ is a $N_b \times N_b$ mutual impedance matrix, which give a matrix relationship between SEP of middle point of conductors $[\bar{\mathbf{V}}_b]$ and leakage currents $[\bar{\mathbf{I}}_s]$ through the rapid Galerkin's Moment method [16].

Combining Eqs. (2) and (3), we have

$$\begin{bmatrix} \bar{\mathbf{F}} \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{\bar{\mathbf{A}}}_{\mathrm{pb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bp}}^t & \bar{\bar{\mathbf{A}}}_{\mathrm{pb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}^t \\ \bar{\bar{\mathbf{A}}}_{\mathrm{bb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bp}}^t & \bar{\bar{\mathbf{A}}}_{\mathrm{bb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}^t + \bar{\bar{\mathbf{Z}}}_s^{-1} \end{bmatrix} \begin{bmatrix} \bar{\mathbf{V}}_p \\ \bar{\mathbf{V}}_b \end{bmatrix} \tag{4}$$

At last we have

$$[\bar{\mathbf{F}}] = [\bar{\bar{\mathbf{Y}}}_b][\bar{\mathbf{V}}_p] \tag{5}$$

$$\begin{aligned} [\bar{\bar{\mathbf{Y}}}_b] = {} & \left[\bar{\bar{\mathbf{A}}}_{\mathrm{pb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bp}}^t\right] \\ & - \left[\bar{\bar{\mathbf{A}}}_{\mathrm{pb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}^t\left[\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}^t + \bar{\bar{\mathbf{Z}}}_s^{-1}\right]^{-1}\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bp}}^t\right] \end{aligned} \tag{6}$$

The vector of nodal SEP $[\bar{\mathbf{V}}_p]$ may be calculated through solving the Eq. (5). The SEP of middle point $[\bar{\mathbf{V}}_b]$ can be calculated by

$$[\bar{\mathbf{V}}_b] = -\left[\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}^t + \bar{\bar{\mathbf{Z}}}_s^{-1}\right]^{-1}\left[\bar{\bar{\mathbf{A}}}_{\mathrm{bb}}\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}\bar{\bar{\mathbf{A}}}_{\mathrm{bp}}^t\right][\bar{\mathbf{V}}_p] \tag{7}$$

The leakage currents $[\bar{\mathbf{I}}_s]$, and the branch currents $[\bar{\mathbf{I}}_b]$ can also be calculated [4–7].

Once branch and leakage currents are known, the electromagnetic field quantities like as SEP φ, A, E and B at any point can be calculated, which can be referred in papers [4–7].

The study of the grounding systems performance in the frequency domain has been reduced to the computation of $[\bar{\bar{\mathbf{Z}}}_s]$ and $[\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}]$ matrices. From Refs. [4–9], we know that each segment is modeled as a lumped resistance and self-inductance. Interactions of mutual inductance or impedance between branch segments' branch or leakage currents are also included in the method. For matrix $[\bar{\bar{\mathbf{Y}}}_{\mathrm{bb}}]$ case, the diagonal elements consist of self impedance and self induction, other elements belong to mutual induction between branch currents along a pairs of conductor segments. The formula for self impedance and self induction can be referred to Refs [4–9]. For matrix $[\bar{\bar{\mathbf{Z}}}_s]$ case, whose element $Z_{i,j}$ is the mutual impedance coefficient between leakage currents along a pair of segments, including conductive and capacitive effects of the earth. It must be pointed out that formula of mutual impedance coefficients is almost same as the one in Refs. [4–9], only different