Journal of Electrostatics 73 (2015) 33–37

Contents lists available at ScienceDirect

Journal of Electrostatics

journal homepage: www.elsevier.com/locate/elstat

Fractal dimension of long electrical discharges

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ARTICLE INFO

Article history:
Received 15 April 2014
Accepted 30 October 2014
Available online 12 November 2014

Keywords:
Fractals
Tortuosity
Long sparks
Edge detection
Image analysis

ABSTRACT

The fractal dimension of 500 mm long electrical discharges is presented by analyzing a set of photographic images. Three popular fractal dimension estimation techniques, box counting, sandbox and correlation function methods were used to estimate the fractal dimension of the discharge channels. To remove the apparent thickness due to varying magnitudes of current in the discharge channels, edge detection algorithms were utilized. The estimated fractal dimensions for box counting, sandbox and correlation function for long laboratory sparks were 1.20 ± 0.06, 1.66 ± 0.05 and 1.52 ± 0.12 respectively. Within statistical uncertainties, the estimated fractal dimensions of positive and negative polarities agreed very well.

Introduction

Estimating the tortuosity of the irregular paths taken by the discharge channels is necessary when determining the extent to which the complex paths contribute to the electromagnetic radiation or even attributing to the type of electrical breakdowns. In the past, a statistical method which was based on the idea of a path taken by a random walker has been utilized to quantify the channel tortuosity of natural lightning channels, triggered lightning channels and long laboratory sparks [1–3]. The main disadvantage of this statistical technique is the need to accurately identify the direction changes of the linear sections of the channel along the path of the discharge channels. The identification could become quite difficult and lead to errors when branches are present with varying degrees of luminosity which may appear to be close to one another in a 2D view, especially for complex discharge channels. It is virtually impossible to apply this statistical method to find the channel tortuosity of surface discharge patterns or electrical trees. Thus, this technique is useful only for discharge channels having no branches or for less complex discharge channels with a few clearly visible branches.

As an alternative, fractal techniques can be applied to characterize the channel tortuosity in electrical discharge channels. Radom and complex patterns have been successfully characterized by using fractal dimension. Niemeyer et al. [4]. have reported a fractal dimension for two-dimensional branched gas discharges as 1.7, which is normally considered as the common value for the fractal dimension of electrical discharges that are highly branched. By analyzing a set of lightning photographs, Tsonis [5] reported a fractal dimension of natural lightning as ~1.34. Most of the recent work in this area has concentrated on the development of stochastic dielectric breakdown models by computer simulations in two-dimensions or three-dimensions to estimate the fractal dimensions of electrical discharges [6–9]. Thus, it is important to document fractal dimensions of long electrical discharges since collaborative experimental measurements are scarce in the literature. It would also be interesting to compare the fractal dimension of triggered lightning flashes with natural lightning flashes given the fact that the reported tortuosity values are significantly different for the two processes [1,2]. However, this is out of the scope of the work presented in this paper.

When estimating fractal dimensions, there are a number of different algorithms that can be applied to electrical discharges [10]. Some of these algorithms depend on the spatial distributions of fractal patterns and others depend on the time-dependent nature of fractal development. Due to the different definitions of the fractal dimension, different values for the same fractal structure could be observed. One may refer the work done by Barclay et al. [11] and Kudo et al. [12] in this area.

In this work, the Box counting, Sandbox and Correlation function methods were utilized to estimate the fractal dimension of 500 mm long electrical discharges. Several popular edge detection algorithms were utilized in pre-processing the photographic images in order to remove the apparent thickness of the discharge channel caused by the magnitude of the flowing currents through different branches which may affect the estimation of fractal

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http://dx.doi.org/10.1016/j.elstat.2014.10.015
0304-3886/

dimensions [4]. The edge detection algorithms were designed using Sobel, Prewitt and Laplace convolution operators.

Methodology

Experiment

All measurements were carried out at the High Voltage Institute of the Uppsala University, Sweden. Long sparks were generated by applying an impulse voltage between a steel rod of 10 mm diameter with a hemispherical tip placed 0.500 m vertically above a circular aluminium plate which acted as a plane electrode using a Marx generator. Three cameras placed symmetrically around the spark gap at 2.00 m distance, captured the discharge events simultaneously. The details of the experimental setup and pre-processing of the photographic images are documented elsewhere [3] and thus, only the essential details relevant to this work is described here.

Fig. 1 shows the placement of the cameras with respect to the spark gap and the coordinate system adopted in this work. Basically, the x–z axis refers to the view of the cameras with z along the spark gap (the dimension perpendicular to the plane of the paper). The y coordinate refers to the depth with respect to the view plane of the cameras. The still photographs of the captured images were carefully scanned, digitized and saved as grey scale TIF images. The data consisted of 18 image sets with each set representing three different camera views (a total of 54 images) taken with four different impulse voltage settings. That is, 4 image sets for +22 kV, 5 image sets for +44 kV, 5 image sets for −44 kV and 4 image sets for −75 kV.

Fractal dimension

In this work, three popular fractal dimension estimating methods were utilized in estimating the fractal dimension of electrical discharges. These three fractal dimension methods utilize different techniques to examine the scaling behaviour of bulk (area) of the fractal pattern with size (length). A brief description of each of the method adopted in this work is given below. The reader may refer the work done by Theiler [10] for a comprehensive description of various fractal dimension estimation methods.

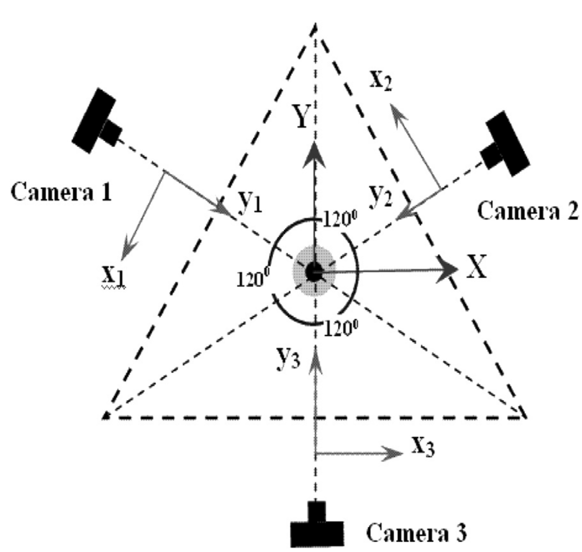

Fig. 1. Camera geometry used in the experimental setup. Cameras were placed symmetrically around the spark gap at 2.00 m distance from the axis of the rod electrode and triggered simultaneously (long shutter time).

(i). Box counting method:

The box counting method is a one of the simplest and popular methods for estimation of fractal dimension. In this method, local scaling is compared with the bulk size. In other words, the number of squares '$n(r)$' which is required to cover the fractal pattern for different sizes of the square 'r' is counted. In implementing the algorithm, one counts the squares without weighting for the number of points of the pattern within each square. The box counting dimension D1 can be represented by [10],

$$r \propto \left[\frac{1}{n(r)}\right]^{D1} \quad (1)$$

(ii). Sandbox method:

Sandbox method is a variant of the box counting method. In the Sandbox method a square of size 'L' is formed on the pattern and the bulk of the tree found within the square is evaluated. The bulk can be evaluated by counting the number of lattice points within the square. Average bulk $M(L)$ is obtained for different squares of sizes 'L'. Fractal dimension D2 is the exponent that expresses the scaling of the bulk with its size [11].

$$M\left(L\right) \propto L^{D2} \quad (2)$$

(iii). Correlation function method:

The correlation function method gives a statistical value for the fractal dimension based on pair-wise distance between points. Basically, the correlation dimension can be calculated between each pair of point in a set of 'N' points. In this method, bulk is equal to the average of the number of pairs where pair distance is less than the defined length 'r'. The bulk can be calculated using the correlation integral [13],

$$C(r) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{j=1}^{N} \sum_{i=j+1}^{N} \Theta(r - |X_i - X_j|)$$

where $(X_i - X_j)$ is the pair-wise distance and Θ is given by

$$\Theta\left(r - \left|X_i - X_j\right|\right) = \begin{cases} 1 \; 0 \le (r - |X_i - X_j|) \\ 0 \; 0 > (r - |X_i - X_j|) \end{cases}$$

The correlation dimension D3 can be calculated mathematically by using

$$C(r) \propto r^{D3} \quad (3)$$

Generally, in all fractal estimation methods, the fractal dimension can be found by calculating the gradient of a double log plot. However, the criterion for choosing the best gradient is not well defined. In general, the longer the range, the better the estimate. The range is limited on one hand by the finite size of the pixels and on the other hand by the finite size of the image.

Edge detection

The apparent thickness of the discharge channels due to the flowing currents in different branches affects the fractal dimension estimates. Thus, image pre-processing is required to identify the boundary of the discharge channels. Ideally, one would expect to restrict the width of the discharge channels to be an infinitesimally