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Current density modeling of a linear pin–plane array corona discharge

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ABSTRACT

In a recent paper, we reported on negative pin-to-plane corona in a quasi-electrostatic approximation. The model was consistent with the experimentally observed variation of corona current with the inverse of the square of gap distance. This paper assesses current density distributions of a linear array of point coronodes by using our Laplacian model of negative point-to-plane system. The results of the modeling are compared to other experimental data on density distributions available in the literature. The trend and shape of the theoretically derived longitudinal current density and those observed experimentally roughly agree. The advantages and limitations of the analysis are also presented.

1. Introduction

Point-to-plane discharges are often studied in correlation with industrial applications of point–plane arrays. Linear arrays of negative point coronating systems are known to create a more uniform longitudinal current than negative wire emitters [1], which typically exhibit intense spots of isolated corona discharge occasionally moving across the wire [2] (positive wire emitters have a more uniform corona current distribution [2]). They also generate less ozone than negative wire–plate systems operating at an equivalent point [3] (nevertheless, negative polarity produces 5–8 times more ozone than positive corona does [4]). Some researchers consider that point arrays also have the advantage of good resistance to mechanical shocks and lower onset voltage [5] compared to wire systems. Various modifications of linear pin arrays are found in practice as tufts, usually studied as micropoints on wires [6]. It is known that discharges from multiple points generate truncation effects in the corona current profiles [6], and there is noticeable interaction between individual point discharges [1,7–9]. It has been reported that the lateral current density distribution is similar to that found for a single pin by Warburg (see Section 1.1), and that the corona current generated obeys the Townsend relation $I = KV(V - V_0)$, where V_0 is the threshold voltage for corona discharge and K is a proportionality constant [1]. From a theoretical point of view, a linear point array with a very large inter-point distance should simulate the corona discharge characteristics associated with independent point–plane discharges. Also, a wire–plane configuration can be considered as a limiting case of a linear point array with an inter-point distance of zero. Therefore, some common characteristics of the coronating point–plane, linear point–plane array, and wire–plane configurations should be observed, along with the distinct behavior of each discharge. The Warburgian lateral current density distribution, defined in Eq. (1), appears to be a typical unifying characteristic.

1.1. Warburgian current density distributions

Warburg was the first to report that current density distribution in a pin-to-plane geometry in air varies with its corresponding vertex angle α according to a $\cos^n \alpha$ law [10] relation. Location of the vertex and the magnitude of the angle α can be easily seen in Fig. 1 if the stressed electrode array shown there is simplified to one corona pin. Warburg empirical law [10,11] for angles smaller than 60° is given by

$$J_W(\alpha) = J_W(0) \cos^n \alpha \tag{1}$$

where $J_W(0)$ is the maximum corona current produced in the pin-to-plane arrangement and the function $J_W(\alpha)$ shows how the corona current intercepted by the ground plane depends on the vertex angle α. For angles larger than the cut-off angle 60°, currents abruptly drop to zero. For a small point-to-plane gap (less than 7 cm), air at normal atmospheric pressure and humidity, Warburg determined the exponent n to be 4.65 for negative corona and 4.82 for positive

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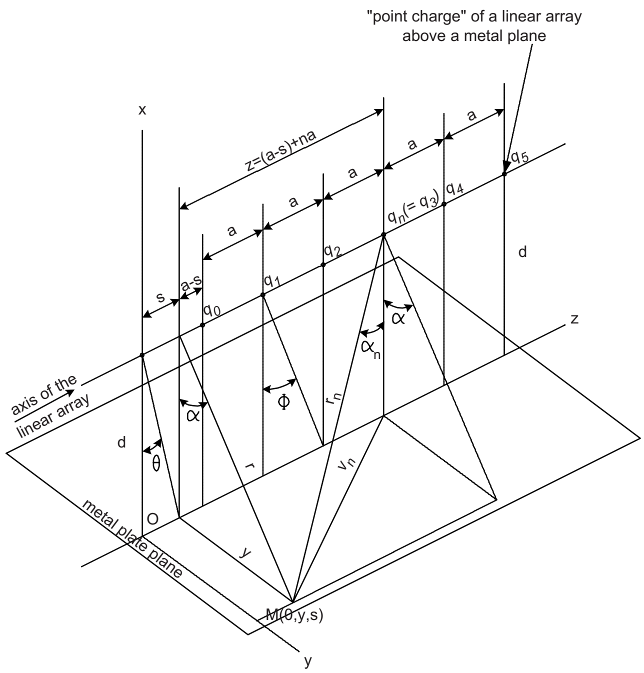

Fig. 1. Linear array of several pin-to-plane systems, separated by distance a.

corona. The Warburg law Eq. (1) has been observed for various gap lengths [12–19], pressures [16,20, p. 235] and special regimes (hysteresis region of negative corona in pure nitrogen) [21]. The exponent n is usually considered to be 5, but various curve fittings of experimental data show variations (values of 4.5–5.5 were found by Goldman et al. [15]; values of 4.8–6 were found by McLean and Ansari [18]). Wire–plane coronating systems also generate Warburg-like current density distribution in the plane perpendicular to the wire [22–26]. The exponent for wire–plane coronas in air ranges from 4.2 to 5.98, according to recent literature [26].

The goal of the present paper is to extrapolate the Laplacian model developed in Ref. [27] to determine the linear point–plane array corona generated current profiles, as well as examine the advantages and limitations of this extrapolation.

2. Linear array of pin-to-plane systems

Current density distribution in a linear array of point corona nodes (coronodes) can be developed from the modeling of a single pin, negative point-to-plane arrangement.

2.1. Laplacian model

A phenomenological relation for both pin–plate and wire–plate corona systems has been found previously by Walsh [19,24]:

$$J = CE_0^m \tag{2}$$

where J is the current density at the plate, E_0 is the Laplacian field at the plate, and C and m are proportionality constants. The Laplacian field can be used, as long as the space charge between the electrodes is reasonably low. In the space between the electrodes, on and near the axis of the pin, the relation is similar to the one given by Sigmond [28] for what he defines as "saturation" current density. In our previous work, this phenomenological relation was used (with $m = 2$) for developing a Laplacian model, assessing the current distribution of negative point–plane systems [27].

The versatile phenomenological relation (Eq. (2)) is of the type published in 1966 by Usinin [29] and used for wire–plane electrodes by Walsh et al. in 1984 [24]. At the plate electrode, the current varies inversely with the square of the pin–plate distance, which is consistent with many but not all experimental observations for negative pin–plane, sharp points, and small gaps [30–32], [16, p. 230], [33, p. 349].

2.2. Modeling a coronating linear pin–plane array

Let us consider a linear array of pins above an infinite conducting plate at distance d. The pins are placed equidistantly with a pin–pin separation a. The pin electrodes are all at the same voltage with respect to the metal plate, and for simplicity are modeled as "point charges" above the plane (see Fig. 1):

$$q_1 = q_2 = q_3 = \ldots = q_n = \ldots = q. \tag{3}$$

In the three-dimensional Cartesian diagram, the conducting plate is located in the y–z plane, and the array of the point charges, separated by equidistant a, is on a line parallel with O–z, intercepting O–x at a distance d from the origin O. Vertical projections of the positions of the point charges are shown on the diagram, intersecting axis O–z on the plane of the conducting plate. Let M be a point situated on the plane at a distance y from axis O–z, and distance s from O–y. The distance from point M to the axis of the linear array of "point charges" is denoted as r. The angle between r and the projection plane of the linear array axis on the metal plane is marked as α. The nth "point charge" q_n has a vertex angle α_n with respect to point M, as shown in Fig. 1. The linear array is also characterized by Φ, which is the angle defined by the perpendicular to the axis plane and a line extending from any one point charge to the point where its nearest neighbor's position projects onto the plane, where

$$\tan \Phi = \frac{a}{d}. \tag{4}$$

An estimate of the current density at point M, derived from the results of the pin-to-plane electrostatic model, is developed as follows. The normal electric field produced by the negative "point charge" q_n at point M is, according to relation (18) in Ref. [27]:

$$E_n = 2k\frac{\cos^3 \alpha_n}{d^2}, \tag{5}$$

where the magnitude of charge q_n is contained in the parameter $k = q_n/(4\pi\varepsilon_0)$. Using the notations in Fig. 1, $\cos \alpha_n$ can easily be derived as

$$\cos \alpha_n = \cos \alpha \frac{1}{\sqrt{1 + \cos^2 \alpha [n \tan \Phi - \tan \theta]^2}}. \tag{6}$$

If the "point charges" to the left of q_1 are numbered as $q_{-1}, q_{-2}, q_{-3}, \ldots, q_{-n}$ then the electric field produced by q_{-n} at point M is $E_{-n} = 2k(\cos^3 \alpha_{-n}/d^2)$ and

$$\cos \alpha_{-n} = \cos \alpha \frac{1}{\sqrt{1 + \cos^2 \alpha [n \tan \Phi + \tan \theta]^2}}. \tag{7}$$

The superposition of the electric field of N "point charges" of the linear array to the right of M (with positive projections on axis O–z) and of N charges to the left of M (with negative projections on axis O–z) yields

$$E_N = \sum_{n=1}^{N} (E_n + E_{-n}). \tag{8}$$