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About behavior of electrostatic pendulum near conducting or dielectric plate

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A R T I C L E I N F O

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A B S T R A C T

The interaction of an electrostatic pendulum with a grounded conducting plate or a dielectric plate has been examined experimentally and theoretically. The force of interaction of a charged ball (an element of the pendulum) with its image in a plate was measured in equilibrium position. It was found that in both cases, for the same set of system parameters, the pendulum can have two stable equilibria. Calculation of the pendulum potential energy has shown that at certain values of the system parameters the potential energy can have two local minima corresponding to the two stable equilibria. A numerical simulation of an equation of bistable pendulum motion at external parametric force showed possibility of chaotic oscillations of the pendulum.

1. Introduction

In this paper, the term "electrostatic pendulum" means a system which consists of a rod with a charged ball on its end that is able to rotate about an axis going through the rod; the system is a physical pendulum modified such that its ball is charged. This kind of pendulum has been used, in particular, as an electrostatic dynamometer to measure electrostatic force [1–5]. In Ref. [1,2,4], the interaction of two charged conducting balls was examined, and their interaction force was measured. In Ref. [3,4], the interaction of a charged ball with a grounded conducting plate was examined, and the force of interaction of the ball with its electrical image was measured. In Ref. [5], some properties of an electrostatic pendulum and other oscillators of this type have been investigated theoretically and experimentally. In Ref. [5], in particular, it was shown that there are bifurcation points in the working range of pendulum–conducting plate system parameters that separate regions with stable pendulum equilibria, those with unstable equilibria, and those with neither of the two. Importantly, for the same set of system parameters, the pendulum can simultaneously have one unstable equilibrium and two stable equilibria. Thereby, an electrostatic pendulum of this type is placed into the category of bistable oscillators which can exhibit mixed and, specifically, chaotic dynamics (see, for example Ref. [6]).

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In this paper, we present some results obtained in Refs. [4,5]. Furthermore, we provide data on the experiments performed to measure the electrical image force in a dielectric (glass) plate. Furthermore, pendulum nonlinear oscillations near the conducting plate at the parametric force causing chaos in the system are examined theoretically (numerically).

As the charged ball motion speeds and accelerations are low and the experimental setup is small in size, in this paper we neglect the electromagnetic radiation effects and the electromagnetic field delay, thereby, staying within the limits of electrostatics.

2. Measurement of electrical image force

The method of electrical images was checked experimentally (see Refs. [3–5]). For this purpose, the force acting on a charged ball located at some distance from the conducting grounded plate was measured. To measure the force, an electrostatic dynamometer consisting of a charged ball on a rotating rod was used (Fig. 1a). The force was proportional to the rod end deflection (A in Fig. 1a), i.e. to the value f measured by a millimeter ruler.

During the first sets of experiments, a grounded duralumin plate $645 \times 380 \times 2\ mm^3$ in size was used. Suspending the plate by a fishing line eliminated such factors as the surface conductivity of the surrounding bodies and the additional induced capacitance, thereby insulating the plate from all bodies except the grounding wire. A foam plastic ball 24 mm in diameter coated with thin aluminum foil was glued to a ballpoint pen rod with a wire from

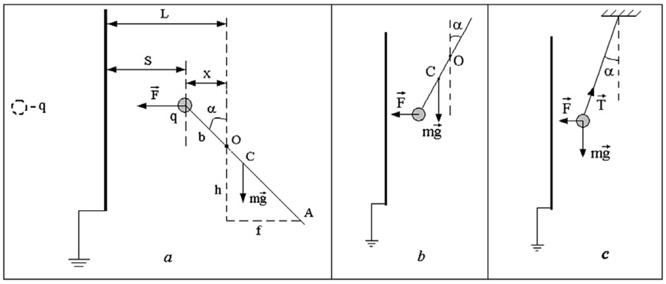

Fig. 1. (a) Schematic diagram of the electrostatic pendulum used in the experiment. (b, c) Schematic diagrams of the other similar pendulums.

high-voltage source passing inside. The Coulomb force can cause the entire system to rotate about the O axis (Fig. 1a). The other setup parameters were as follows: OC = d = 5.0 mm, b = 60 mm, h = 100 mm, m = 1.1 g. A voltage of 15 kV was applied to the ball from a high-voltage source through an insulated lead.

When the pendulum is in its equilibrium position, the moment of the Coulomb force about the O axis is balanced by the moment of the entire system force of gravity (Fig. 1a):

$$Fb\cos\alpha = mgd\sin\alpha, d = \text{OC}. \quad (1)$$

From here, we can obtain:

$$F = \left(\frac{mgd}{bh}\right)f. \quad (2)$$

Assuming the ball to be a point-like charge (the assumption is proved in Ref. [4]), the Coulomb force acting on it from the full image is

$$F = \frac{kq^2}{4S^2}, \quad k = \frac{1}{4\pi\varepsilon_0}, \quad (3)$$

where ε_0 is an electric constant, q is a charge of the ball. The distances L, b, h and f (Fig. 1a) were measured by a millimeter ruler, and the distance S was calculated from

$$S = L - x = L - \frac{bf}{\sqrt{h^2 + f^2}}, \quad (4)$$

Based on the experimental data, the force F (2), acting on the ball from the side of the electrical image in the plate, was plotted as a function of the inverse square of the ball center–plate distance in the stable equilibrium position (see Fig. 2, each data point represents at least five measurements). The straight line in the figure corresponds to the Coulomb's law. For each initial ball position shown (L = 8.0; 8.2; 8.3 cm), there are two corresponding stable equilibrium positions of the pendulum – on the left and on the right from the shaded region. In this shaded region, there are (according to theory – see below) no stable equilibria for the pendulum, nor has experiment revealed any.

Fig. 3 shows two equilibrium positions of the pendulum near conducting grounded plate, which correspond to the same set of the system parameters $Q^2 \approx 0.28$, $L \approx 12$ and the voltage of 15 kV. The angles of the rod displacement from the vertical line are $\alpha_a \approx 18°$, and $\alpha_b \approx 80°$.

The angles of the rod displacement from the vertical line are $\alpha_a \approx 18°$, $\alpha_b \approx 80°$.

The experimental results obtained suggest that in a sufficiently large, thin grounded conducting plate, the full charge image $q' = -q$ is formed and that in the parameter range studied, the charge–source interaction force obeys the Coulomb's law, as indeed it should. A quite unexpected finding was that the conducting plate–electrostatic pendulum system has the bistability property, i.e., can have two stable equilibria simultaneously for the same set of parameters.

3. Pendulum equilibrium theory

It is easy to see that the problem of the equilibrium of the pendulum in Fig. 1a is equivalent to the problems in Fig. 1b and c. Indeed, the equilibrium condition for the pendulum in Fig. 1b is identical to Eqs. (1) and (2), and that for the pendulum in Fig. 1c has the form $F/mg = \tan\alpha$ and can be reduced to Eq. (2) by renormalizing the constants m or g.

In the sequel, we have $kq^2b/4S^2 = mgd\tan\alpha$. Using Fig. 1a, we obtain $S^2 = (L - b\sin\alpha)^2 = b^2(\tilde{L} - \sin\alpha)^2$, where $\tilde{L} = L/b$. Now, the equilibrium equation in the dimensionless form can be written as follows:

$$Q^2 = \left(\tilde{L} - \sin\alpha\right)^2 \cdot \tan\alpha, \quad Q^2 = \frac{kq^2}{4mgdb}. \quad (5)$$

The obtained transcendent equation serves to determine the angular displacement of the pendulum from equilibrium as a function of two parameters, Q^2 and L. As algebraic equations are easier to deal with, we eliminate the angle using the equality $\sin\alpha = \tilde{L} - \tilde{S}$, where $\tilde{S} = S/b$. The tilde "∼" is omitted henceforth. Then, the equilibrium equation becomes:

$$Q^2 = \frac{S^2(L-S)}{\sqrt{1-(L-S)^2}}. \quad (6)$$

Eq. (6) serves to determine ball center–plate distance S, sustained in equilibrium as a function of two parameters, L and Q^2. The equation is difficult or impossible to solve for S analytically, but it allows, for example, expressing L explicitly as a function of S, with Q left as a parameter. After some algebra, we find that

$$L = S + \frac{Q}{\sqrt{S^4 + Q^4}}. \quad (7)$$

Fig. 4 presents dependence $L(S)$ calculated for different values of Q^2 using Eq. (7). Curves 1, 2, 3 correspond to Q^2 = 0.05; 0.28; 0.6. For a given value of L on curve 2, there are three values of S that correspond to equilibrium equation. However, point B cannot be a stable equilibrium because decreasing L here corresponds to

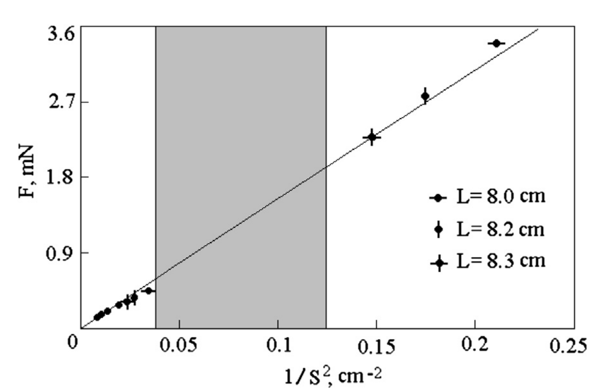

Fig. 2. Force acting on the ball from the side of the electrical image in the plate as a function of the inverse square of the distance between the ball center and the plate. For each initial ball position shown, there are two corresponding stable equilibrium positions of the pendulum – on the left and on the right from the shade region. The straight line is the Coulomb's law.