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Three-dimensional static Green's function for a half space over mixed PEC and dielectric wedges

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ABSTRACT

A pseudo-analytical solution technique is proposed to determine the three-dimensional static Green's function for a half space over mixed perfect electric conductor-dielectric wedges. The governing Poisson's equation is solved, using the Kontorovich–Lebedev and Fourier transforms. The solution, expressed in terms of image contributions, consists of an excitation point source, a set of point images in physical space, and two line images located in complex space for each region of the problem geometry. The validity of the proposed technique is confirmed by comparing our results with the existing analytical solutions and those obtained numerically using the using a finite integration code.

1. Introduction

Many electric power installations such as wind turbines, wave energy converters, and transmission lines are located near shorelines [1–3]. Near field electromagnetic analysis of the apparatus in these installations are of great interest in electromagnetic compatibility (EMC) studies, particularly when they are exposed to nearby cloud-to-ground lightning electromagnetic fields. The main issue in these studies involves the inhomogeneous nature of sea-land geometry. Although the analysis of sea-land geometry goes back to the works of Millington [4], Kirke [5], and Wait [6,7], these methods involve the far field approximation and cannot analyze the near field electromagnetic components.

The static solution is a fundamental component of any dynamic problem. The dominant behavior of a solution to the Helmholtz equation can be extracted from the static solution in the neighborhood of boundary discontinuities and sources [8–11]. The modified image theory introduced in Ref. [12] is a quasi-static approach to analyze electromagnetic fields in planar geometries. The validity of the quasi-static method has been demonstrated in Ref. [13] for the near field region in the vicinity of a homogenous ground.

The quasi-static approach for computation of electromagnetic fields in the vicinity of sea-land geometry involves solution of the governing Poisson's equation. A simple model of sea-land geometry considering the sea section as a perfect electric conductor (PEC) half plane has been proposed by Clemmow [14]. The sea section fulfills the assumption of perfect electric conductivity to a sufficient degree of accuracy. A more accurate model can be described by a half space mixed PEC and dielectric wedges, representing the sea and land sections, respectively.

A solution for the two-dimensional Poisson's equation in the vicinity of a dielectric wedge is presented in Ref. [15]. In that work, the exact solution of the static dielectric wedge problem is obtained via the Mellin transform. The three-dimensional (3D) Poisson's equation in the presence of a dielectric wedge has been studied by Nikoskinen et al. [16]. The solution is expressed in terms of image sources using the Kontorovich–Lebedev and Fourier transforms. The use of image sources including distributions in complex space provides an efficient method to compute the potential inside and outside the wedge. Another treatment of the static dielectric wedge resulting in residue series is presented in Ref. [17].

This paper presents an exact solution to the 3D static Green's function for a half space over mixed PEC and dielectric wedges. The determination of the Green's function serves a systematic approach for treating quasi-static problems involving sea-land geometry. The solution is based on the extension of the wedge static Green's

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function [16,17]. It uses an eigen function expansion using the Kontorovich–Lebedev and Fourier transforms. To achieve numerical efficiency, the resultant potential function is expressed in terms of potential functions due to appropriate point and line image sources.

The manuscript is organized as follows. In Section 2, first, the boundary value problem representing a point source of unit magnitude in a half space over mixed PEC and dielectric wedges is formulated. The closed-form contributions due to the PEC and dielectric wedges are then derived to determine the respective 3D static Green's function. In Section 3, solutions are presented for special cases for which the analytical solutions are available in the literature. For further validation of the proposed solution, simulation results are given where they are compared with those obtained using the finite integration technique.

2. Theory

2.1. Formulation of the boundary-value problem

The geometry of the problem associated with the 3D static Green's function for a half space over mixed PEC and dielectric wedges is shown in Fig. 1. As shown in this figure, a point source of unit magnitude is located at $(\rho_0, \varphi_0, 0)$ in cylindrical coordinates in a half space over mixed PEC and dielectric wedges. The dielectric constants of the surrounding medium and dielectric wedge are ε_1 and ε_2, respectively. The interface of PEC and dielectric wedges is located at $\varphi = \gamma$ where the potential solution according to $0 < \gamma < \pi/2$ is considered in this paper. The external potential in the source region Φ_1, satisfies Poisson's equation,

$$\nabla^2 \Phi_1(\rho, \varphi, z) = -\frac{1}{\varepsilon_1 \rho}\delta(\rho - \rho_0)\delta(\varphi - \varphi_0)\delta(z), \quad (\pi/2 \le \varphi \le 3\pi/2), \tag{1}$$

and the internal potential in the dielectric wedge Φ_2, satisfies Laplace's equation,

$$\nabla^2 \Phi_2(\rho, \varphi, z) = 0, \quad (-\pi/2 \le \varphi \le \gamma). \tag{2}$$

The boundary conditions at the interface of the dielectric wedge and the surrounding medium and the Dirichlet boundary conditions at the PEC wedge surfaces are

$$\Phi_1(\rho, 3\pi/2, z) = \Phi_2(\rho, -\pi/2, z), \tag{3}$$

$$\varepsilon_1 \frac{\partial}{\partial \varphi}\Phi_1(\rho, 3\pi/2, z) = \varepsilon_2 \frac{\partial}{\partial \varphi}\Phi_2(\rho, -\pi/2, z), \tag{4}$$

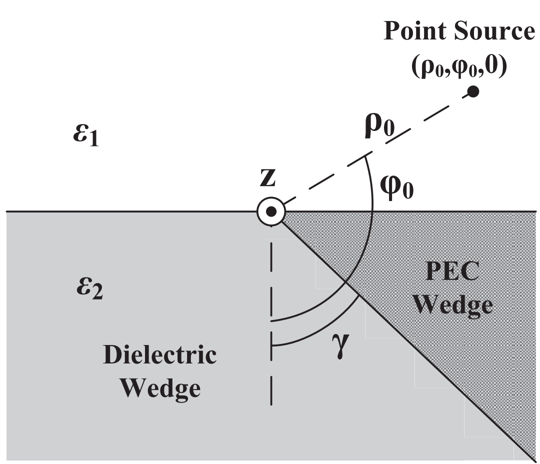

Fig. 1. Cross-sectional (plane $z = 0$) view of a point source in a half space over mixed PEC-dielectric wedges.

$$\Phi_1(\rho, \pi/2, z) = 0, \tag{5}$$

$$\Phi_2(\rho, \gamma, z) = 0. \tag{6}$$

Using the multiplication of Eq. (1) by ρ^2, *i.e.*,

$$\left(\rho^2\partial^2/\partial\rho^2 + \rho\partial/\partial\rho + \partial^2/\partial\varphi^2 + \rho^2\partial^2/\partial z^2\right)\Phi_1(\rho, \varphi, z) = -\frac{\rho}{\varepsilon_1}\delta(\rho - \rho_0)\delta(\varphi - \varphi_0)\delta(z), \tag{7}$$

additional two conditions can be written at the point source as

$$\Phi_1\left(\rho_0, \varphi_0^+, 0\right) = \Phi_1\left(\rho_0, \varphi_0^-, 0\right), \tag{8}$$

$$\frac{\partial}{\partial \varphi}\Phi_1\left(\rho_0, \varphi_0^+, 0\right) - \frac{\partial}{\partial \varphi}\Phi_1\left(\rho_0, \varphi_0^-, 0\right) = -\frac{\rho}{\varepsilon_1}\delta(\rho - \rho_0)\delta(z). \tag{9}$$

A Fourier cosine transform in z and its inverse as

$$G(k) = \int_0^\infty \cos kz\, g(z)\mathrm{d}z, \quad g(z) = \frac{2}{\pi}\int_0^\infty \cos kz\, G(k)\mathrm{d}k, \tag{10}$$

and a Kontorovich–Lebedev transform in ρ and its inverse as [18]

$$F(k,\upsilon) = \int_0^\infty K_\upsilon(k\rho) f(k,\rho)\frac{\mathrm{d}\rho}{\rho}, \; f(k,\rho) = \frac{1}{\pi i}\int_{-i\infty}^{i\infty} I_\upsilon(k\rho) F(k,\upsilon)\upsilon\,\mathrm{d}\upsilon, \tag{11}$$

can be used to expand the solution to Eqs. (1) and (2) in terms of a complete set of basis functions. It should be mentioned that the Fourier cosine transform with respect to z is utilized due to the even symmetry of the problem geometry with respect to the z coordinate.

The eigen functions of the Laplace's equation are the modified Bessel functions of the first kind, the sine and cosine functions, and the cosine functions with respect to ρ φ, and z coordinates, respectively. Therefore, the unknown potential can be presented as follows,

$$\Phi_2(\rho, \varphi, z) = \mathscr{L}[a(\upsilon)\sin\, \upsilon\varphi + b(\upsilon)\cos\, \upsilon\varphi], \quad (-\pi/2 \le \varphi \le \gamma), \tag{12}$$

$$\Phi_1^+(\rho, \varphi, z) = \mathscr{L}[c(\upsilon)\sin\, \upsilon\varphi + d(\upsilon)\cos\, \upsilon\varphi], \quad (\pi/2 \le \varphi \le \varphi_0), \tag{13}$$

$$\Phi_1^-(\rho, \varphi, z) = \mathscr{L}[e(\upsilon)\sin\, \upsilon\varphi + f(\upsilon)\cos\, \upsilon\varphi], \quad (\varphi_0 \le \varphi \le 3\pi/2), \tag{14}$$

where $\mathscr{L}[.]$ denotes an integral operator as

$$\mathscr{L}[\Psi(\upsilon,\varphi)] = \frac{1}{\pi^2 i}\int_0^\infty \cos\; kz \int_{-i\infty}^{i\infty} I_\upsilon(k\rho)\, K_\upsilon(k\rho_0)\Psi(\upsilon,\varphi)\, \mathrm{d}\upsilon\mathrm{d}k. \tag{15}$$

The representation for the Dirac-delta functions using the Kontorovich–Lebedev and Fourier transforms is

$$\rho\delta(\rho - \rho_0)\delta(z) = \frac{1}{\pi^2 i}\int_0^\infty \cos\; kz \int_{-i\infty}^{i\infty} I_\upsilon(k\rho)\, K_\upsilon(k\rho_0)\upsilon\, \mathrm{d}\upsilon\mathrm{d}k. \tag{16}$$