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Estimation of capacitance of different conducting bodies by the method of rectangular subareas

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Abstract

The capacitance evaluation of arbitrary-shaped conducting bodies is an important step for the estimation of spacecraft equivalent circuit model for the prediction of electrostatic discharge. In this paper, an attempt has been made for the evaluation of charge distribution and hence the capacitance of arbitrary-shaped conducting surfaces. Surfaces are modeled by planar rectangular subdomains in which the charge density is assumed to be constant. The exact formulation for the matrix element is evaluated for rectangular subsection. The Method of Moments with pulse basis function and point matching is employed to calculate the charge distribution on the surface and hence the capacitance. This paper presents the results for capacitance of different conducting shapes, e.g., square, rectangular, circular, annular circular disk, T-shaped, L-shaped, triangular, annular triangular, etc. The results have been compared with other available results in literature wherever possible.

Keywords: Electrostatic discharge; Electrostatic capacitance; Conductors; Rectangular subdomain; Method of Moments

1. Introduction

There has been considerable interest in the evaluation of the capacitance and charge distribution of different conducting structures such as rectangular plates, square plates, circular and annular discs, hollow cylinder, etc., located in free space because their use in spacecraft. The capacitance evaluation has an important application for the determination of spacecraft equivalent circuit models for the prediction electrostatic discharge. The earliest work on the evaluation of capacitance of square plate appears to have been carried out by Maxwell [1]. Smythe had derived closed form/approximate expressions for computing capacitance of a few conducting objects such as bowl, cylinder, circular disc, etc. [2]. There are some interesting publications on the capacitance calculation of three-dimensional multiconductor systems and different high voltage electrode configurations [3–4]. In the work of Ruehli and Brennan, the basic equations for the potential coefficients of rectangular conducting element were derived and used for the evaluation of capacitance of square plate, cube via Method of Moments [3]. However, the resulting equations for the potential coefficients are found to be complicated and also these were mainly used for two-/three-dimensional bodies with square/rectangular surfaces [3]. In Ref. [4], the capacitance to ground of different high voltage electrode configurations was evaluated, neglecting the influence of other grounded or live structures. Also different methods of calculation were used for different electrode geometries, e.g., sphere, toroid, cylinder, circular disc, rectangular plate, and box [4]. However, the authors had not noticed any work on the evaluation of capacitance of arbitrary planar conducting bodies with a more generalized and simple elemental shape, which can be used for any planar surface. Harrington evaluated data on the capacitance of a square conducting plate employing square subdomain regions, but did not present clearly the exact formulas for

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the evaluation of the matrix elements for the evaluation of capacitance [5]. The triangular subdomains had been used for more complex surfaces by Rao et al. [6]. Also, Chakraborty et al. [7] had obtained a method of computing the capacitance of a cylinder and a truncated cone by employing cylindrical subsections. In the present paper, the authors have concentrated on the numerical evaluation of capacitance of conducting objects in free space using Method of Moments and rectangular subdomain modeling. The rectangular shape is chosen because of its ability to conform easily to any geometrical surface or shape and at the same time maintaining the simplicity of approach compared to the triangular patch modeling. Here, the exact formulation for the evaluation of the impedance matrix for rectangular subdomain is determined. The results are compared with other available data in literature.

2. Formulation

We consider a perfectly conducting surface is charged to a potential V. The unknown surface charge density distribution $\sigma(r')$ may then be determined by solving the following integral equation:

$$V = \int\int_S \frac{\sigma(r')}{4\pi\varepsilon|r - r'|}\,\mathrm{d}s'. \tag{1}$$

Here, r and r' are the position vectors corresponding to observation and charge source points, respectively, $\mathrm{d}s'$ is an element of surface S and ε is the permittivity of free space. The exact solution for the charge distribution can be obtained only for a few very specialized geometries. In the general case, the surface is discretized and the charge distribution is found by solving Eq. (1) using numerical methods. Here, the arbitrary-shaped bodies are approximated by planar rectangular subdomains (Fig. 1). The Method of Moments with pulse basis function and point matching is then used to determine the approximate charge distribution. On each subdomain, a pulse expansion function $P_n(r)$ is chosen such that $P_n(r)$ is equal to 1 when r is in the nth rectangle and $P_n(r)$ is equal to 0 when r is not in the nth rectangle. With the above definition of expansion function, the charge density, $\sigma(r')$ may be approximated as

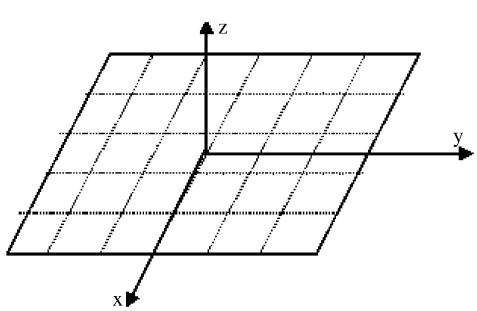

Fig. 1. Square plate divided into rectangular subsections.

follows:

$$\sigma(r') = \sum_{n=1}^{N} \sigma_n P_n(r') \quad \text{where } P_n = \begin{cases} 1 & \text{for } n\text{th subsection} \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

Here, N is the number of rectangles modeling the surface and σ_n's are the unknown weights (charge density).

Substitution of charge expansion (2) in (1) and point matching the resulting functional equation, by enforcing equality of the two sides of the equation for observation points located at the center of each rectangle, yields an $N \times N$ system of linear equations which may be written in the following form

$$[V] = [K][Q]. \tag{3}$$

Here, $[K]$ is an $N \times N$ matrix and $[Q]$ and $[V]$ are column vectors of length N.

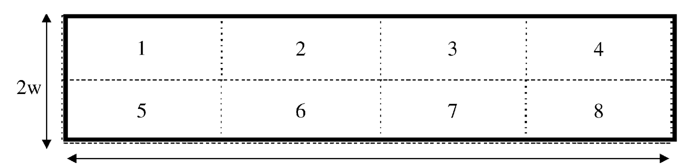

Fig. 2. Rectangular plate ($2L = 4$ m; $2w = 1$ m; $V = 1$ V) divided into 4×4 subsections; capacitance = 54.73 pF.

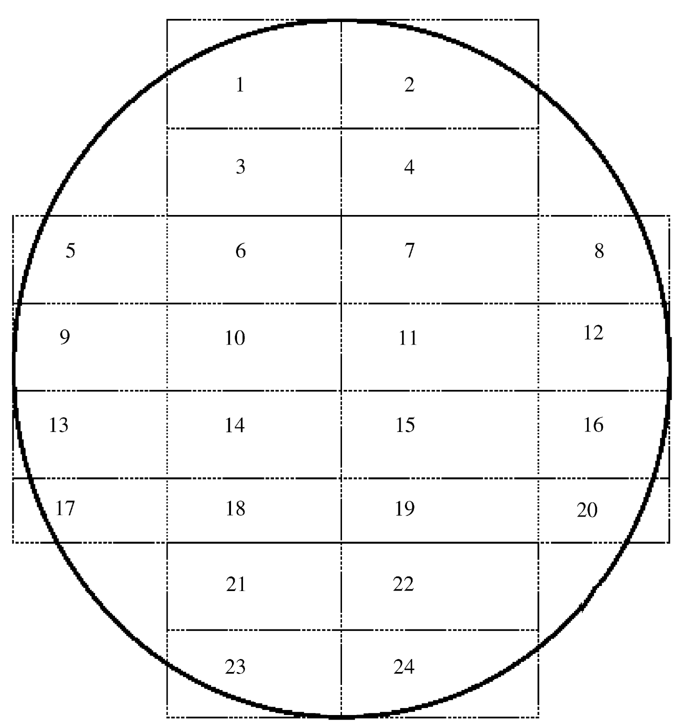

Fig 3. Circular disc (radius = 1 m, $N = 24$); capacitance = 68.36 pF agrees with analytical value = 70.73 pF [11].