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A mathematical model of external electrostatic field of a special collector for electrospinning of nanofibers

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ABSTRACT

This article focuses on the analysis of electrostatic field generated by a special collector consisting of two parallel cylindrical conductors used for electrospinning. Computed values of critical electrical potential obtained by an analytical model are compared with measurements. Here we show that the gap distance between the special collector and a spinning electrode, i.e., a syringe needle, has not substantial effect on the critical potential if the special collector distance from the syringe needle is higher than the gap distance between cylindrical conductors. The presented model is a good analytical approximation of electric field created by the used special collector.

1. Introduction

Over the past few years electrospinning of polymer solutions has gained much attention mainly as an inexpensive and simple method for the laboratory and industrial production of polymer nanofibers [1]. The polymer nanofibers are fibers with diameters in scale under one micrometer and relatively large surface area per unit mass. Those properties have enabled the fibers to be used, or beginning to be used, for filtration applications, biomedical applications, drug delivery systems, tissue engineering, production of protective clothing and, not least, as a reinforcement of composite materials [2].

Electrospinning is a process in which polymer nanofibers are formed by the creation and elongation of an electrified liquid jet of a polymer solution [3,4] or a polymer melt [5,6] when this jet is exposed to a strong external electrostatic field. This external electrostatic field plays the important role, because it is the cause of the stretching and accelerating of an electrically charged liquid jet and it also stabilizes its straight part to a certain distance from a syringe needle or the other spinneret [2]. Usually an electrical potential difference in the range of tens of kilovolts is applied between a syringe needle and a grounded collector. Due to the external electrostatic field between the syringe needle and the grounded collector an electric charge is induced on the surface of a polymeric droplet. A hemispherical surface of the polymeric droplet at the orifice of the syringe needle is gradually extended. A cone shape, the so-called Taylor's cone, is formed with a time lag [7–9]. By further increasing the voltage supplied by the high voltage source, a certain critical field strength, E_c, is reached at the tip of Taylor's cone, which corresponds to the critical voltage V_c. While the critical field, E_c, is the material characteristics for the specific polymer solution, the critical voltage, V_c, is spinner's geometry dependent parameter. Thereafter an electrical pressure, which is due to electrostatic forces, overcomes a capillary pressure [10]. Collapse of Taylor's cone simultaneously occurs and an electrically charged liquid jet is ejected [11]. The jet has bending and whipping instabilities due to repulsive forces between the surface charges. These instabilities generate a looping and spiraling path [2,12–14] and it leads to the extreme elongation and acceleration of the electrically charged liquid jet [3]. This mechanism, although it was discovered almost one century ago, is not fully understood yet [13].

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During the elongation of the electrified liquid jet, the jet surface area increases dramatically, such that up to 90% of a solvent is evaporated [15] and almost dry polymer nanofibers are collected on the grounded collector.

Special collectors of various shapes play the key role in case of electrospinning when the targeted orientation of deposited nanofibers is required. Presented work was motivated by experiments with them, specifically by collector made from two parallel cylindrical rods of finite diameter, that were previously published [5,16–18]. This collector gives rise to the parallel nanofibers arrangement, which are perpendicular to the axes of collector cylinders. However, such design has been studied experimentally without any analytical description of the electric field. A mathematical description of electric field distribution for this special collector arrangement is the main aim of our paper. Referred formulas are useful for the enhancement of numerical models of the electrospinning process, which computes the path of an electrified liquid jet. Calculated critical voltage, V_c, is further experimentally verified.

2. Material and methods

2.1. Materials

Poly(vinyl alcohol), PVA, was used as 10 wt % solution in water. Density and surface tension of the solution were $\rho = 1006.9$ kg/m^3 and $\gamma = 46.29$ mN/m, respectively. The experiments were done under ambient conditions at room temperature 26 °C and relative humidity of about 56%.

3. Theoretical background

An electrostatic field inside a spinning chamber can be considered as a system consisting of a conductive droplet of a polymer solution and a special collector (see Fig. 1). The origin of the Cartesian coordinate system is identified with the orifice of a syringe needle. For simplicity the special collector is considered as a set of two parallel, infinitely long, cylindrical conductors with linear charge density q_τ.

Under those assumptions, the problem can be converted to two-dimensions. Electrostatic field of two parallel conductors of the same finite diameters is modeled using solution for the electric field of two infinitely thin conductors having equipotential planes corresponding to the surfaces of both conductors of finite diameter. The conformal mapping could be used for the solution [19]. This transformation allows to find an analytical function that satisfies prescribed boundary conditions. The complex potential F is, in case of two charged infinitely thin conductors, located symmetrically around the axis z, defined by formula

$$F = j\frac{q_\tau}{2\pi\varepsilon_0\varepsilon_r}\ln(w+e) + j\frac{q_\tau}{2\pi\varepsilon_0\varepsilon_r}\ln(w-e) + C, \tag{1}$$

where j is the imaginary unit, q_τ is the linear charge density, ε_0 is the permittivity of free space, ε_r is the relative permittivity of a surrounding medium (i.e. air), $w = x + jz$ is the complex number, e is the gap distance between the axes of infinitely thin conductors and C is the constant of integration.

Let us consider complex numbers expressed in an exponential form

$$\begin{aligned} w + e &= \left|\mathbf{r} - {}^1\mathbf{r}'\right| \exp(j\alpha_1), \\ w - e &= \left|\mathbf{r} - {}^2\mathbf{r}'\right| \exp(j\alpha_2), \end{aligned} \tag{2}$$

where $\mathbf{r} \equiv \{x; z\}$ is the position vector of a point in the complex plane, ${}^1\mathbf{r}' \equiv \{-e; h\}$, ${}^2\mathbf{r}' \equiv \{e; h\}$ are the position vectors of the axes of infinitely thin conductors and h is the distance between the orifice of the spinneret and the special collector. Substituting Eq. (2) into Eq. (1) we can get the complex potential, F, in the following formula

$$F = -\frac{q_\tau}{2\pi\varepsilon_0\varepsilon_r}(\alpha_1+\alpha_2) + j\frac{q_\tau}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\left|\mathbf{r}-{}^1\mathbf{r}'\right|\left|\mathbf{r}-{}^2\mathbf{r}'\right|\right) + C.$$

We denote the real part Φ and imaginary part φ of the complex potential, F, respectively

$$\Phi = -\frac{q_\tau}{2\pi\varepsilon_0\varepsilon_r}(\alpha_1+\alpha_2) + C_1,$$

$$\varphi = \frac{q_\tau}{2\pi\varepsilon_0\varepsilon_r}\ln\left(\left|\mathbf{r}-{}^1\mathbf{r}'\right|\left|\mathbf{r}-{}^2\mathbf{r}'\right|\right) + C_2, \tag{3}$$

where Φ is the electric flux, φ is the electrical potential and $C_1 + C_2 = C$ are the constants of integration.

The next step is to investigate a distribution of equipotential surface $\varphi = \text{const}$. The equipotential surfaces are given from Eq. (3) by formula

$$\left|\mathbf{r}-{}^1\mathbf{r}'\right|\left|\mathbf{r}-{}^2\mathbf{r}'\right| = \text{const}.$$

They are the geometric locus of points, where the product $|\cdot||\cdot|$ of their distances from two fixed points is constant. The curve that satisfies this property is called Cassinian oval [20] and it is described by equation

$$\sqrt{(x+e)^2 + (z-h)^2}\sqrt{(x-e)^2 + (z-h)^2} = k^2.$$

Constants e and k^2 are chosen so that Cassinian oval best approximates surface of circular conductors, which should be the equipotential plane to meet the field boundary conditions on the surface of conductors. However, only four intersections of circular conductors with Cassinian oval can satisfy this. Let us choose their coordinates as $[-a/2 - R; h]$, $[-a/2 + R; h]$, $[a/2 - R; h]$, $[a/2 + R; h]$, where a is the gap distance between the axes of two parallel conductors and $R = D/2$ is their diameter. Thus

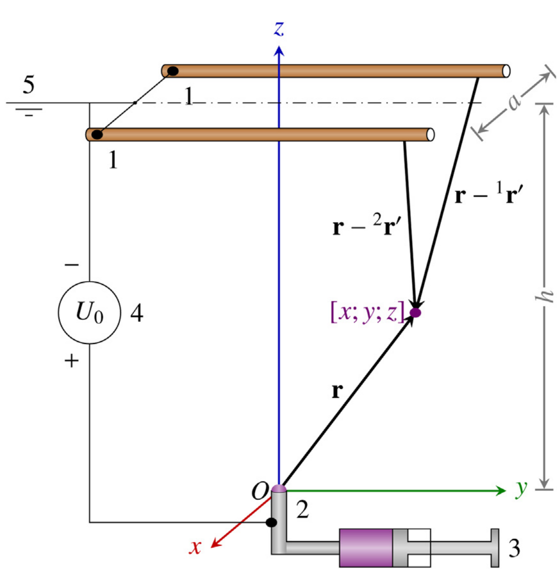

Fig. 1. Schematic illustration of electrospinning setup. 1—conductive electrode (two copper rods with the gap between them), 2—spinning electrode with polymeric droplet, 3—pump for feeding polymer solution, 4—high voltage source, 5—grounding.