Journal of Electrostatics 71 (2013) 591–596

Contents lists available at SciVerse ScienceDirect

Journal of Electrostatics

journal homepage: www.elsevier.com/locate/elstat

Modeling static charge dissipation on solids: An historical perspective

Philippe Molinié

Département Energie, Supélec, 91192 Gif-sur-Yvette Cedex, France

A R T I C L E I N F O

Article history:
Received 10 September 2012
Received in revised form
13 November 2012
Accepted 4 December 2012
Available online 20 December 2012

Keywords:
History of Electrostatics
Relaxation
Charge transport
Voltage decay
Dielectric models

A B S T R A C T

Gilbert was the first to recognize the specific character of *electrics*, materials able to attract a needle when rubbed. Four centuries later, the detailed understanding of this experiment remains delicate, even concerning one aspect only: charge dissipation after charging. The double nature of the material, dielectric and allowing charge transport, identified by Faraday, will participate. We examine here both aspects, following an historical perspective. Dielectric absorption, involving slow polarization mechanisms, can be related to a viscoelastic behavior of the material, as long as superposition principle applies. From Kohlrausch to modern spectroscopy, dielectric functions were proposed, and attempts were made to account for the general behavior, involving time power laws and stretched exponentials.

Charge transport in insulating solids may be modeled using the concepts of carrier mobility and trapping. In disordered materials, dispersive transport has to be considered, due the broad distribution in trapping energies. This leads also to time power laws in the decay process. Hence both faces of the insulator, dielectric and conductive, often lead to the same dispersion in the time response of the signal. It may be related to intrinsic parameters of the material, like its fractal nature. It has also important practical consequences.

1. Introduction

The basic experiment in Electrostatics consists in charging a material (for instance, by rubbing) and checking what phenomena are involved by this action. This experiment is the founding landmark of the discipline, when in the end of the 16th century, William Gilbert (Fig. 1) used a pivoting needle to undertake a systematic study of the forces produced by many materials, thus establishing the scientific field of *electricity* (he introduced the word) as distinct from *magnetism* [1]. The purpose of this paper is to discuss one aspect concerning this simple experiment, which is the way that charge will dissipate on the solid. Charge generation mechanisms will not be discussed here. The focus will be put on the time response of the insulator (for instance, the voltage time curve). Following an historical perspective, it will be shown that four centuries of scientific research were not enough to exhaust the topic.

In his pioneering 1837 experiments, Michael Faraday (Fig. 2) set the stage for this complexity. Using a charged Leyden jar as a generator, a Coulomb torsion balance as a charge measurement device, and a spherical capacitor made of a small brass sphere mounted within a hollow larger one to test various liquids, gas or solids (Fig. 3) he discovered and described the electric charge induction phenomenon, which we now call permittivity [2]. Hence he recognized that insulators, whose basic property was to oppose a resistance to an electric current, were also dielectrics. From that time insulating materials have two faces, as Janus, the double-headed roman god of doors and doorways, grasping a walking stick in one hand and a key in the other. We will examine in the next two sections these two faces, which may be involved in the time response. Interestingly, they lead in the end to the same kind of time responses involving time power laws.

2. Dipoles in motion, from Leyden jars residual voltage to dielectric spectroscopy

2.1. Delayed response, residual voltage and superposition principle

One of the interesting remarks that Faraday noted carefully in its treaty is the following:

When this apparatus was first charged with electricity up to a certain intensity, as 400°, measured by the Coulomb's electrometer, it sank much faster from that degree than if it had been previously charged to a higher point, and had gradually fallen to 400°; or than it would do if the charge were, by a second application, raised up again to 400°: all other things remaining the

E-mail address: philippe.molinie@supelec.fr.

0304-3886/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.elstat.2012.12.001

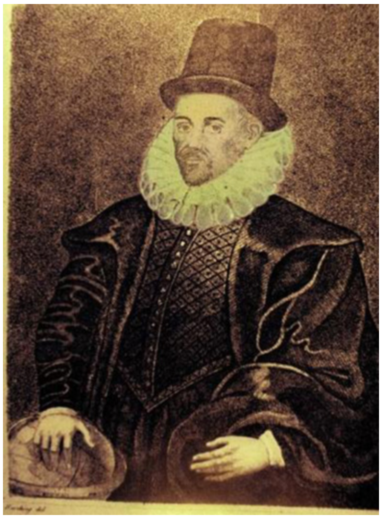

Fig. 1. William Gilbert (1544–1603).

same. Again, if after having been charged for some time, as fifteen or twenty minutes, it was suddenly and perfectly discharged, even the stem having all electricity removed from it, then the apparatus being left to itself, would gradually recover a charge, which in nine or ten minutes would rise up to 50° or 60°, and in one instance to 80° [2, note n°1234].

He describes here the memory effect of the insulator, responsible also for the observed residual voltage. Mathematical tools to describe precisely this phenomenon were developed forty years later. John Hopkinson, an engineer and mathematician working for the leading glass manufacturer in England, presented in 1876 a communication to the Royal Society aiming to model residual voltages on Leyden jars. It was published with a note from Maxwell signaling the link between Hopkinson calculations and the theory of viscoelasticity published two years before by Boltzmann. Hopkinson took into account this remark and issued later a more detailed model of the insulator dielectric response based on the superposition principle [3]. In the frame of these theories, the insulator is fully described by a linear dielectric function. Voltage and charge are related by the following (modern) relations:

Fig. 2. Michael Faraday (1791–1867).

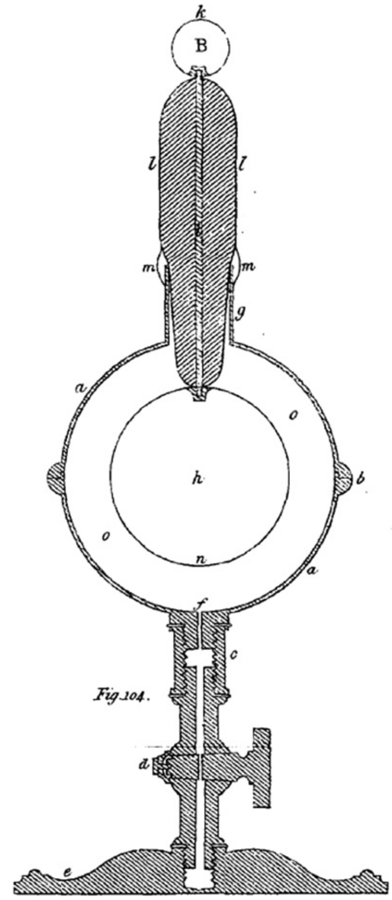

Fig. 3. Faraday's spherical capacitor [2].

$$q(t) = \frac{\varepsilon_0}{d} \int_{-\infty}^{t} V(\tau)\phi_D(t-\tau)\mathrm{d}\tau \qquad (1)$$

$$V(t) = \frac{d}{\varepsilon_0} \int_{-\infty}^{t} q(\tau)\phi_E(t-\tau)\mathrm{d}\tau \qquad (2)$$

ϕ_D and ϕ_E are non-independent dielectric functions, the product of their Laplace transforms being 1.

After charging and disconnecting a capacitor, the voltage will decrease, even without any conduction or charge transport. It may be viewed as the consequence of a progressive increase of the permittivity. When, after a given charging period, a short circuit is performed during a limited time, the residual voltage can be computed from the dielectric function ϕ_E using (2). Absorption and resorption currents flowing when a DC voltage step is applied to the capacitor which is then short-circuited, may be treated in the same way using ϕ_D.

2.2. The shape of the dielectric response and the corresponding voltage decay

The first proposal on the mathematical shape of voltage relaxation (and hence of the dielectric functions) came in 1854, when Rudolf Kohlrausch used the stretched exponential function to account for relaxation effects of a discharging Leyden jar [4]. The first systematic investigations were later made by Jacques Curie. He