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Propagation effects of a fractal rough ocean surface on the vertical electric field generated by lightning return strokes

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ABSTRACT

Based on an improved two-dimension (2D) fractal model of rough ocean surface, the propagation effects of the rough ocean surface on the vertical electric fields generated by lightning return strokes are analyzed. The results show that the rough ocean surface has much effect on the electric field derivatives, but has no or little effect on the field peaks. The frequency above 10 MHz is attenuated significantly by the rough ocean surface, and the rapid attenuation of frequency above 10 MHz in the experimentally obtained spectrum may be taken into account the errors introduced by the roughness of the ocean surface.

1. Introduction

The electromagnetic fields generated by lightning return strokes have submicrosecond components which play a major role in the interaction of these electromagnetic fields with structures such as power line [1–3]. The high frequency components undergo a rapid attenuation when they propagate over the finitely conducting ground [4–10]. Therefore, the experimental investigations on the high frequency components in lightning-generated electromagnetic fields are usually conducted under maritime conditions where the propagation path of the electromagnetic fields is over seawater with a good conductivity of 4 S/m. Weidman et al. [11,12] and Willett et al. [13] found that the experimentally obtained spectrum is rapidly attenuated when the frequencies are higher than about 10 MHz, and they extended the frequency spectrum generated by lightning return strokes up to 20 MHz. In theory, Ming and Cooray [14] analyzed the propagation effect of the rough ocean surface on the lightning electromagnetic fields, and found that the rough ocean surface may have much effect on the propagation of the lightning high frequency components. However, the rough ocean model that they used is Newmann–Pierson spectrum as proposed by Kinsman [15], this model is out of date and is a semi-empirical sea-spectrum model, and it has much defect and is only used for the approximate engineering application.

Since natural surfaces are generally neither purely random nor purely periodic and often anisotropic, the introduction of fractal geometry provided a new tool for describing natural rough structures [16]. A normalized two-dimension band-limited Weierstrass fractal function shows a combination of both deterministic periodic and random rough structures, and this function is usually used to model an anisotropic and rough ground surface [17–21]. However, the Weierstrass fractal function is a fixed model and cannot be used to simulate any random rough ocean surface.

Therefore, based on an improved two-dimension (2D) fractal rough ocean surface model [22], we will analyze the propagation effects of the roughness of the ocean surface on the lightning vertical electric fields, because this model is more accurate to depict the fine structure of the rough ocean spectrum, and the simulated results may be more reliable. The fractal spectrum of this model satisfies the positive and negative power law spectrum when the spatial wave numbers are smaller and larger than the fundamental wave number, respectively [22]. The fractal spectrum of this model shows a good agreement with Pierson–Moskowitz (PM) spectrum for the different ocean wind speed.

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2. Cooray approximation for the propagation of the lightning-generated vertical electric field along the rough ground

The ground is considered as a perfectly conducting plane, the lightning channel is straight and vertical to the ground, and the vertical electric field in time domain at any point on the ground level can be expressed as follow [23,24].

$$\begin{aligned}E(r,t) = {} & \frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\theta(z')}{R^3(z')}\int_{t_b}^{t} i(z',\tau-R(z')/c)\mathrm{d}\tau\mathrm{d}z' \\ & +\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\theta(z')}{cR^2(z')}i(z',t-R(z')/c)\mathrm{d}z' \\ & -\frac{1}{2\pi\varepsilon_0}\int_0^{L'(t)}\frac{\sin^2\theta(z')}{c^2R(z')}\frac{\partial i(z',t-R(z')/c)}{\partial t}\mathrm{d}z' \end{aligned} \quad (1)$$

where $t_b(z')$ is the time at which the return stroke current pulse front is “seen” by an observer on the ground, $t_b(z') = (z'^2+r^2)^{1/2}/c$. $L'(t)$ is the length of the channel which an observer “sees” at time t, $t = L'(t)/v+(L'(t)^2+r^2)^{1/2}/c$, v is the return stroke speed, c is the velocity of light, and $i(z',t)$ is the return stroke current distribution along the channel, the other parameters are shown in Fig. 1.

The first term of equation (1) is the electrostatic field component, the second term is the induction field component, and the third term is the radiation field component. According to Cooray approximation [6], the vertical electric field in frequency domain on the finitely conducting ground level is:

$$\begin{aligned}E_{v,\sigma}(j\omega,0,d) = {} & E_{s,\infty}(j\omega,0,d)+E_{i,\infty}(j\omega,0,d) \\ & +E_{r,\infty}(j\omega,0,d)W(j\omega,0,d)\end{aligned} \quad (2)$$

where $E_{v,\sigma}(j\omega,0,d)$ is the total vertical electric fields in frequency domain on the finitely conducting ground, $E_{s,\infty}(j\omega,0,d)$, $E_{i,\infty}(j\omega,0,d)$ and $E_{r,\infty}(j\omega,0,d)$ are the field components in frequency domain corresponding to the electrostatic field, the induction field and the radiation field shown in equation (1), respectively, assuming the ground to be perfectly conducting. The three field components in frequency domain can be solved by using a Fourier Transforms routine from equation (1).

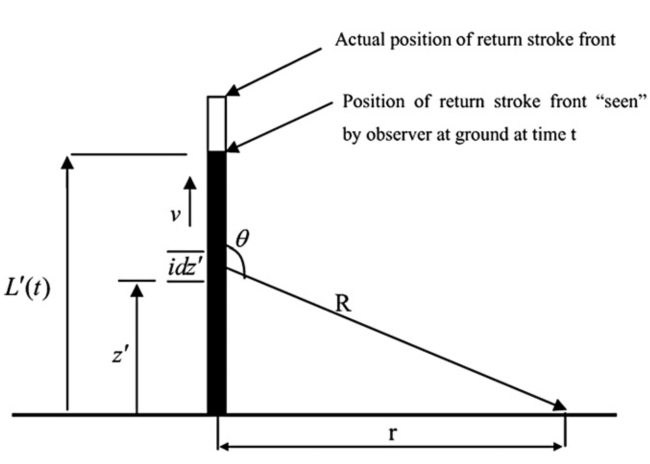

Fig. 1. Geometry used in deriving the expressions for the vertical electric fields generated by lightning return strokes on the ground level.

$W(j\omega,0,d)$ is the attenuation function corresponding to a dipole located at the lower end of the channel. The attenuation function $W(j\omega,0,d)$ is given by Wait [25–27]:

$$W(j\omega,0,d) = 1-j\sqrt{\pi p}\exp(-p)\mathrm{erfc}(j\sqrt{p}) \quad (3)$$

$$p = -\frac{j\omega d}{2c}\Delta_{\mathrm{eff}}^2 \quad (4)$$

where Δ_{eff} is the effective surface impedance corresponding to the rough ocean surface, “erfc” is the complementary error function, d is the horizontally observed distance, ω is the radial frequency, c is the light speed, $j = \sqrt{-1}$. Therefore, from equations (1)–(4), we can see that the effective surface impedance Δ_{eff} is crucial for the propagation effect of the rough ocean surface on the lightning electric fields.

3. An improved two-dimension (2D) fractal rough ocean surface model

An improved 2D fractal ocean surface model is given as [22]:

$$\begin{aligned}f(x,y,t) = {} & \tau\chi\sum_{m=0}^{N-1}a^{-(d-\xi)m}\sin\{k_0a^m[(x+v_xt)\cos\beta_{1m} \\ & +(y+v_yt)\sin\beta_{1m}]-\Omega_{1m}t+\Phi_{1m}\} \\ & +\tau\chi\sum_{m=0}^{N-1}b^{(d-3)m}\sin\{k_0b^m[(x+v_xt)\cos\beta_{2m} \\ & +(y+v_yt)\sin\beta_{2m}]-\Omega_{2m}t+\Phi_{2m}\}\end{aligned} \quad (5)$$

where τ is the standard deviation of ocean wave height, and $\tau = 0.0212\cdot\zeta\cdot v_{19.5}^2/4$, ζ the correction factor, $v_{19.5}$ the wind speed at 19.5 m height above the ocean surface. χ the normalize constant, N the iteration number, $2<d<3$ the fractal dimension, ξ is the positive exponential correlation factor. a the scale factor when the spatial wave number is less than the fundamental frequency, b the scale factor when the spatial wave number is larger than fundamental frequency, k_0 the fundamental spatial wave number. v_x, v_y the Radar motion velocity in x direction and y direction, respectively, β_{1m} and β_{2m} the wave propagating directions, Ω_{1m} and Ω_{2m} the angular frequencies, Φ_{1m} and Φ_{2m} the random wave phases which evenly distribute in $[-\pi, \pi]$. The normalize constant χ is given as:

$$\chi=\left\{\frac{2\left[1-a^{-2(d-\xi)}\right]\left[1-b^{2(d-3)}\right]}{\left[1-a^{-2(d-\xi)N}\right]\left[1-b^{2(d-3)}\right]+\left[1-a^{-2(d-\xi)}\right]\left[1-b^{2(d-3)N}\right]}\right\}^{1/2} \quad (6)$$

The improved 2D fractal ocean surface model is composed of a series of sine function with variable amplitude and periodic, random wave phases, it shows a combination of both deterministic periodic and random rough structures, and is a good candidate for modeling natural surfaces.

Fig. 2 shows the simulated 2D fractal rough ocean surface by using equation (5). Where $v_{19.5}=10$ m/s, $\zeta=1.698$, $\tau=0.9$ m, $b=1.015$, $a=1/b$, $d=2.62$, $\xi=3.9$, $t=20$ s, $N=400$, $v_x=15$ m/s, and $v_y=15$ m/s. The arrow shows the assumed direction of the electromagnetic field propagation.

In order to calculate the effective surface impedance Δ_{eff} of the improved 2D fractal rough ocean surface, the average height density spectral corresponding to this model has to be firstly solved. Wang et al. [22] found by some complicated mathematical operations: