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Spatial scaling, proportional thinking, and numerical understanding in 5- to 7-year-old children

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A B S T R A C T

The present study investigated the role of spatial scaling and proportional-reasoning skills in children's number-line estimations. Proportional strategies in number-line estimations might suggest that correlations between number-line knowledge and scaling are driven by proportional thinking. However, analyses of data on spatial scaling, proportional reasoning, counting skills, and number-line knowledge from 5- to 7-year-old children (N = 65) showed significant correlations between spatial-scaling performance and number-line knowledge, even after age, counting skills, common method variance, and proportional reasoning were accounted for. Thus, spatial scaling is related to variance in number-line estimations due to mechanisms beyond proportional reasoning. The ability to mentally transform magnitudes may be the additional common underlying process.

1. Introduction

Numerous studies have indicated that spatial thinking and numerical reasoning are closely related (for reviews, see Mix & Cheng, 2012; Newcombe, Levine, & Mix, 2015). This link has been shown in several cross-sectional studies (e.g., Casey, Nuttall, Pezaris, & Benbow, 1995; Hegarty & Kozhevnikov, 1999; Kyttälä, Aunio, Lehto, van Luit, & Hautamäki, 2003; Mix et al., 2016; Reuhkala, 2001). Furthermore, several longitudinal studies have demonstrated that spatial skills predicted young children's performance on mathematical tasks (e.g., Gunderson, Ramirez, Beilock, & Levine, 2012; Lauer & Lourenco, 2016; Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2017) as well as adolescents' later career choices and success in the STEM (Science, Technology, Engineering, and Mathematics) disciplines (Kell, Lubinski, Benbow, & Steiger, 2013; Shea, Lubinski, & Benbow, 2001; Wai, Lubinski, & Benbow, 2009).

Much of this research on associations between spatial skills and mathematical performance has focused on mental rotation, showing that children's and adults' ability to rotate objects in their minds is related to their math scholastic assessment test (SAT) scores (Casey et al., 1995), number-line estimations (Gunderson et al., 2012), geometry concepts (Kyttälä & Lehto, 2008), arithmetic (Reuhkala, 2001), and early numeracy (Kyttälä et al., 2003). However, there are of course many spatial skills other than mental rotation. Yet to date, surprisingly little is known about them and their specific relations to mathematical reasoning.

One spatial skill that deserves a closer look is spatial scaling, or the ability to relate distances in one space to distances in another space (Frick & Newcombe, 2012; Huttenlocher, Newcombe, Vasilyeva, 1999; Vasilyeva & Huttenlocher, 2004). Scaling is an important component of map reading, and children and adults seem to use analog mental transformation strategies for spatial scaling (Möhring, Newcombe, & Frick, 2014; Möhring, Newcombe, & Frick, 2016) akin to mental rotation (Shepard & Metzler, 1971). That is,

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they mentally shrink or expand spatial information in the sense of zooming in or out (of the map), thereby internally transforming magnitude information. Such transformation can be thought of as a holistic strategy that changes the entire image, rather than transforming and comparing image-internal distances and their spatial relations. Importantly, this ability to zoom in or out of a space may be crucial for understanding and mentally transforming other kinds of magnitudes as well, such as numerical magnitudes. That is, children with higher abilities in visualizing and flexibly transforming spatial information (i.e., zooming) may be at advantage when they have to transform numerical magnitudes. These transformations of numerical magnitudes are relevant for several mathematical procedures such as when being asked to divide a number by 4 in a division problem.

Support for the notion that spatial scaling may be relevant for mathematical reasoning comes from previous studies that have shown that scaling and proportional thinking are closely related (Boyer & Levine, 2012; Möhring, Newcombe, & Frick, 2015). An understanding of scale relations may be similar to an understanding of proportional equivalence. That is, just as the relations of internal distances *within* a map and within the real-world space are the same despite differences in absolute size *between* the map and the real-world space, the relations of components in equivalent proportions (e.g., 3/5 and 6/10) are the same, despite differences in absolute sizes. Thus, the ability to comprehend the *relation* of two magnitudes regardless of their absolute sizes may be driving the correlation between spatial-scaling and proportional-reasoning tasks. However, it is still an open question whether spatial scaling is also related to magnitude processing other than proportional reasoning, or how this relation extends to the processing of numerical magnitudes.

Therefore, in the present study, we assessed whether and how spatial scaling is related to performance in tasks that require thinking about numerical magnitudes and involve proportional judgments. In one of these tasks, we presented participants with different relative amounts of water and cherry juice and asked them to judge the taste of these mixtures on a rating scale. Thus, the task did not involve numerical entities; however, it required to encode proportional relations, and to reproduce equivalent relations in a different spatial format. This task had been successfully used in 4- to 10-year-olds in previous studies (Möhring et al., 2015; Möhring, Newcombe, Levine, & Frick, 2016).

In another task, participants were presented with a physical line and asked to indicate where different numbers belong on this line. This number-line task has often been used in previous studies with children and adults (e.g., Barth & Paladino, 2011; Booth & Siegler, 2006; Landy, Silbert, & Goldin, 2013; Opfer & Siegler, 2007; Siegler & Opfer, 2003; Slusser, Santiago, & Barth, 2013; Thompson & Opfer, 2010; Rips, 2013) and children's accuracy in locating the numbers was found to be a strong predictor for mathematical performance and learning (Booth & Siegler, 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Gunderson et al., 2012, LeFevre et al., 2013; Siegler & Booth, 2004). If participants locate the numbers correctly with respect to the magnitude each number stands for, their responses would be a linear function of number size. However, young children's estimations typically show a non-linear relation to the normative positions. Some studies have indicated logarithmic response patterns (e.g., Siegler & Opfer, 2003). Other studies have indicated patterns suggesting that children's familiarity with numbers (Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008) or knowledge of the decimal system (Moeller, Pixner, Kaufman, & Nuerk, 2009) influenced their responses. Yet, other research has suggested that even seemingly linear patterns may in fact be best described by one- and two-cycle power models, suggesting a proportional judgment or mental subdivision of the number line (Barth & Paladino, 2011; Slusser et al., 2013).

These latter findings are of crucial importance for the present study because they show that solving this number-line task may require similar proportional judgments as required in the proportional-reasoning and spatial-scaling tasks. Thus, in the present study, we analyzed individual children's response patterns to clarify the role of proportional thinking in the number-line task, and to further investigate its potential contribution to the correlation between scaling and number-line performance. To this end, we tested whether children would indeed use proportional judgments in the number-line task (as indicated by one- or two-cycle functions) or produce patterns in accordance to a linear or logarithmic number representation, or an unbounded power function (which would indicate that children focus on the origin of the number line only).

To measure spatial-scaling abilities, we used a spatial-localization task similar to the ones that had been used in previous research (Frick & Newcombe, 2012; Huttenlocher et al., 1999; Vasilyeva & Huttenlocher, 2004). The task consisted of two conditions. In one condition (non-scaled), children had to simply reproduce a target location presented on a map in a same-sized referent space. In another condition (scaled), the same referent space was presented, but the maps were smaller, such that children had to mentally scale spatial magnitudes to match the referent space. This manipulation was implemented to investigate the specificity of possible correlations. If correlations were due to scaling abilities, we expected them to only show in the scaled version of the task (cf. Möhring et al., 2015). In contrast, if correlations were driven by some surface similarities between tasks (spatial nature, response mode, etc.), one could expect the correlations to be similar for scaled and non-scaled trials.

Finally, to control for individual differences in children's general familiarity with numbers, we assessed their counting abilities. Counting reflects which numbers children have been introduced to and their understanding of the ordinal structure of the symbolic number system. However, in contrast to number-line estimations, it does not necessarily require an understanding of numerical magnitude. We used a counting-on task, in which the experimenter starts to count and the child is asked to count on beyond the next decade change to avoid reciting of rote memorizations (e.g., Ebersbach et al., 2008). As this task can be solved without having a sense for numerical magnitudes and does not involve proportional judgments, we did not expect counting to correlate with proportional reasoning or spatial scaling. However, we expected a correlation between children's counting and number-line estimations because both tasks require an understanding of whole numbers (cf. Aunola, Leskinen, Lerkkanen, & Nurmi, 2004).

The goals of the present study were two-fold. First, we intended to investigate the relation between spatial scaling and number-line estimations. Based on the previous research outlined above, one could assume that these two abilities are connected, as both seem to involve proportional judgments. As a second goal, we investigated whether spatial scaling explained any additional variance