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People's conditional probability judgments follow probability theory (plus noise)

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ABSTRACT

A common view in current psychology is that people estimate probabilities using various 'heuristics' or rules of thumb that do not follow the normative rules of probability theory. We present a model where people estimate conditional probabilities such as $P(A|B)$ (the probability of A given that B has occurred) by a process that follows standard frequentist probability theory but is subject to random noise. This model accounts for various results from previous studies of conditional probability judgment. This model predicts that people's conditional probability judgments will agree with a series of fundamental identities in probability theory whose form cancels the effect of noise, while deviating from probability theory in other expressions whose form does not allow such cancellation. Two experiments strongly confirm these predictions, with people's estimates on average agreeing with probability theory for the noise-cancelling identities, but deviating from probability theory (in just the way predicted by the model) for other identities. This new model subsumes an earlier model of unconditional or 'direct' probability judgment which explains a number of systematic biases seen in direct probability judgment (Costello & Watts, 2014). This model may thus provide a fully general account of the mechanisms by which people estimate probabilities.

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1. Introduction

A *conditional probability* $P(A|B)$ represents the chance that some event A will occur, given that some event B has definitely occurred. People estimate and use conditional probabilities very frequently in everyday life (for example, when I see dark clouds on the horizon and conclude, given those clouds, that rain is likely later). These probabilities are also central to critical decision making (for example, when a lawyer estimates the chances of winning or losing a case given a piece of evidence, and so decides whether or not to proceed to trial). Indeed, conditional probabilities play a fundamental role in many aspects of learning, reasoning, inference, and decision making under uncertainty. But how do people estimate the probability $P(A|B)$, given their knowledge about A and B? What mental processes underlie people's estimation of conditional probabilities?

Researchers have examined people's conditional probability judgment in various different ways. Perhaps the best-known approach involves presenting people with a kind of mathematical problem where they are given numerical values for the probabilities $P(A), P(B)$ and $P(B|A)$ and then asked to estimate the conditional probability $P(A|B)$ (with their answers compared with the normatively correct value from probability theory). Well known examples of this approach are Eddy's 'breast cancer' problem (described in Gigerenzer & Hoffrage, 1995) and Kahneman and Tversky's 'taxi-cab' problem (Kahneman & Tversky, 1982). These studies reveal various reliable errors and biases in people's manipulation of presented probabilities: people tend to erroneously neglect the base rate $P(A)$, and have a tendency to confuse the conditional probabilities $P(A|B)$ and $P(B|A)$ (the 'inverse fallacy'). However, because these studies ask people to estimate one conditional probability $P(A|B)$ in terms of another conditional probability $P(B|A)$ (leaving the source of $P(B|A)$ unexplained) they tell us little about how conditional probability judgments arise from people's knowledge of events A and B.

These 'mathematical problem' studies suggest that people are extremely poor at assessing conditional probabilities. This fits with a currently dominant view of people's probabilistic reasoning, which is that

> In making predictions and judgments under uncertainty, people do not appear to follow the calculus of chance or the statistical theory of prediction. Instead they rely on a limited number of heuristics which sometimes yield reasonable judgments and sometimes lead to severe and systematic errors (Kahneman & Tversky, 1973, p. 237).

This 'heuristics and biases' view has a level of popularity rarely seen in psychology (with Kahneman recieving a Nobel Prize in part for this work) and has had a major impact in a number of areas (Ariely, 2009; Bondt & Thaler, 2012; Camerer, Loewenstein, & Rabin, 2003; Eva & Norman, 2005; Gigerenzer & Gaissmaier, 2011; Hicks & Kluemper, 2011; Kahneman, 2011; Shafir & Leboeuf, 2002; Sunstein, 2000; Williams, 2010). Studies which directly investigate people's conditional probability judgment for events they have experienced, however, report results that in many ways contradict these findings. In these studies, people are not given a set of probabilities $P(A), P(B)$ and $P(B|A)$ and asked to estimate the conditional probability $P(A|B)$ from those values; instead people are simply asked to estimate probabilities such as $P(A), P(B), P(B|A)$ and $P(A|B)$ from their own experience with the events in question. These studies show low rates of occurrence of base-rate neglect and the 'inverse fallacy' (for a detailed review of these results see Koehler (1996); for a more general discussion of this 'description-experience gap', see Hertwig & Erev (2009)). More recent studies also suggest that people's conditional probability estimates can closely follow the normative rules of probability theory in some ways while deviating from those rules in others. For example, in a recent study Fisher and Wolfe (2014) found that the addition form of Bayes' rule

$$P(A|B)P(B) - P(B|A)P(A) = 0$$

held reliably in people's probability estimates, just as required by probability theory. Zhao, Shah, and Osherson (2009), by contrast, found that the requirement

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$