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Computation of scale elasticity in presence of exogenously fixed inputs

Mahdi Mirjaberi, Reza Kazemi Matin *

Department of Mathematics, Karaj Branch, Islamic Azad University, Karaj, Iran

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ABSTRACT

In this paper, the notion of scale elasticity is initially developed for models with non-discretionary inputs. A new unified formulation, resembling those for classic developments is proposed for computation of scale elasticity in presence of exogenously fixed inputs in the framework of date envelopment analysis.

In order to remove the harshness of fixed inputs, we invoke regression of fixed inputs over efficiency measures and therefore units with more favorable environment indices are discounted for pioneers set. This naturally implies that different production possibility sets are induced according to environmental affect. Scale elasticity is then computed using indirect approach for both differentiable and non-differentiable cases.

Also, the approach is illustrated with numerical examples and proved its benefit over selective approach, proposed by Podinovski and Førsund, by comparing and contrasting with numerical data. Moreover, the geometry and rationale behind the notion are described.

1. Introduction

Data Envelopment Analysis (DEA) is a nonparametric and powerful technique to estimate the production frontier and assess the relative efficiencies of multi-input multi-output production units. In many applications of DEA, certain of the input variables may not be under the direct control of management. In the original DEA, presented by Charnes et al. [2] and Banker et al. [3], these non-discretionary (ND) inputs were excluded or treated as normal discretionary inputs, which may lead to a biased view of efficiency.

Non-discretionary inputs has been widely studied since its advent by Banker and Morey. Since then, plenty of models have been proposed by different authors on the subject of efficiency analysis in presence of fixed inputs. These approached can be referred to in general from two respects: single stage models and multiple stage models.

Banker and Morey [4,5] were the first who have accounted for differences in non-discretionary inputs. Nowadays, their approach can be considered a standard technique for the inclusion of non-discretionary inputs in DEA. Since then, a number of different approaches have been developed to deal with non-discretionary inputs. These approaches can be classified into two major categories by their procedure: Single and Multi-stage methods.

Single stage methods accounts for non-discretionary inputs as exogenously fixed variables. Banker and Morey [4] were the pioneer in this field. They modified the constraints on the non-discretionary inputs within the BCC model. This model differs from the original DEA model by breaking the link between non-discretionary inputs and efficiency. However, Ruggiero [6] showed that this model does not properly restrict the reference set. In

* Corresponding author.
E-mail addresses: mahdi.mirjaberi@kiau.ac.ir (M. Mirjaberi), rkmatin@kiau.ac.ir (R.K. Matin).

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essence, the presence of non-discretionary inputs leads to different frontiers.

To control for these fixed variables, Ruggiero [6] proposed to account for environmental affect in efficiency analysis. His model resembles the one presented by Lovell et al. [7]. In this model (hereafter referred to as the Lovell and Ruggiero model), they added constraints to exclude decision making units (DMUs) with a less favorable production environment. In this model, a unit is allowed as a reference only if it belongs to the same or more favorable production environment than the assessed unit. Otherwise, it is excluded from the reference set.

Lovell and Ruggiero model explicitly restricts the comparison set to include DMUs that face a more favorable environment. Similar to the Banker and Morey model, this model requires a priori specification of the continuous non-discretionary variables. However, Ruggiero [8] showed that as the number of continuous fixed factors increases, the probability of identifying a DMU as efficient by default increases. This ignores comparisons between a given DMU and another DMU that overall, has the same or worse environment even though it has a more favorable level of at least one non-discretionary input. This fact suggests an inherent weakness of the Lovell and Ruggiero model.

Both the Banker and Morey and the Lovell and Ruggiero models control for exogenous variables within DEA. Unfortunately, both methods have inherent weaknesses that has been already overcame in Ruggiero [8] motivated by an alternative two-stage approach developed by Ray [9]. In the first stage, Ray had solved the original BCC model regardless of non-discretionary inputs. This produces the measure FS_o for the unit under evaluation. However, this measures captures both the technical inefficiency and the harshness of the production environment. Therefore, it cannot be employed as the pure technical efficiency.

To remove the harshness of production environment, Ray [9] had suggested to regress the first stage index FS_o over the non-discretionary inputs. In fact, Ray [9] had decomposed the first stage index FS_o into environmental affect and the technical efficiency:

$$FS_o = \text{Environmentalaffect} + \text{technicalefficiency}$$

Muñiz et al. [10] compared several methods for controlling for exogenous variables by simulating a production process and varying the number of non-discretionary factors. The performance of the methods was compared according to the rank correlation and mean absolute deviation (MAD) between estimated and true efficiencies. The results indicated that the three-stage model of Ruggiero [8] performed best in nearly all model scenarios and was the only model robust to sample size and the number of non-discretionary variables. This approach would be treated in more detail in the next section (Section 2).

Regardless of advantages and disadvantages of each method, all these methods focus only on technical nor structural efficiency, that is, extracting inputs or augmenting outputs in order to project an inefficient unit onto the frontier.

However, there is still a missing link between technical and structural efficiencies in efficiency analysis of models with non-discretionary inputs: scale elasticity. Scale elasticity is a differential characteristic that can help determine the benefits corporate planning. In the literature, scale elasticity is defined as the ratio of marginal productivity to the average productivity. More precisely, it is a function of inputs and outputs that measures the maximal changes in inputs or outputs relative to a marginal changes in other inputs and/or outputs, for a given point on the efficient frontier to remain efficient.

Although scale elasticity has been considered widely in the literature, less or almost no attention has been paid to it in presence of fixed inputs. A reason for this shortcoming is the assumption of fully equal proportionality in conventional contributions. There, the output changes are measured regarding the changes in all inputs by the same scaling factor.

Incorporating non-discretionary inputs has brought broader applications to DEA models. However, this evolution was at the cost of modifying the essential axiom of total convexity for production possibility sets (PPS). This can be regarded as another weakness for developing scale elasticity for models with non-discretionary inputs.

This paper incorporates scale elasticity with DEA models in presence of non-discretionary inputs. To amend the partial convexity, scale elasticity would be computed regarding the frontiers of the same or less favorable production environment.

Overall, this paper contributes to the field from several respects: The first, invoking layered efficiency analysis, as proposed by Ruggiero [6], enables us to develop the notion of scale elasticity in presence of fixed inputs; the second, it provides implementation of our approach in the framework of data envelopment analysis; the third, given formulations resemble those in conventional methods, that is, the approach is a natural extension for conventional developments; and the last, comparing and contrasting by numerical examples proves the benefits of our development over present viewpoints.

From managerial viewpoint, scale elasticity gives the totally responsive share of the independent factors in construction of dependent factors. However, in real situations, managers often face with independent factors beyond their discretion of which play a pivotal role in decision making.

For example, in efficiency analysis of an industry,[1] fixed costs[2] and variable costs[3] are respectively non-discretionary and discretionary inputs and total revenue is the only output. An enterprise manager is interested to check the share of variable costs in total revenue. (Please see case 3.) Here, scale elasticity of 0.5455, for instance, means that an increase of variable costs by 100% provokes an increase of total revenue by 54.55%. This may be due to the market downturn.

[1] Group of enterprises operate in the same market as in car industry.

[2] Costs that remain constant throughout the relevant range and often include rent, machinery, buildings, etc.

[3] Cots that vary with outputs throughout the relevant range and often include labors, utility bills, raw materials, etc.