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Toward exact number: Young children use one-to-one correspondence to measure set identity but not numerical equality

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ABSTRACT

Exact integer concepts are fundamental to a wide array of human activities, but their origins are obscure. Some have proposed that children are endowed with a system of natural number concepts, whereas others have argued that children construct these concepts by mastering verbal counting or other numeric symbols. This debate remains unresolved, because it is difficult to test children's mastery of the logic of integer concepts without using symbols to enumerate large sets, and the symbols themselves could be a source of difficulty for children. Here, we introduce a new method, focusing on large quantities and avoiding the use of words or other symbols for numbers, to study children's understanding of an essential property underlying integer concepts: the relation of exact numerical equality. Children aged 32–36 months, who possessed no symbols for exact numbers beyond 4, were given one-to-one correspondence cues to help them track a set of puppets, and their enumeration of the set was assessed by a non-verbal manual search task. Children used one-to-one correspondence relations to reconstruct exact quantities in sets of 5 or 6 objects, as long as the elements forming the sets remained the same individuals. In contrast, they failed to track exact quantities when one element was added, removed, or substituted for another. These results suggest an alternative to both nativist and symbol-based constructivist theories of the development of natural number con-

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cepts: Before learning symbols for exact numbers, children have a partial understanding of the properties of exact numbers.

1. Introduction

Number is one of the core competences of the human mind (Carey, 2009; Dehaene, 1997; Dehaene & Brannon, 2011). From birth, human infants discriminate between sets on the basis of number (Feigenson, Dehaene, & Spelke, 2004; Izard, Sann, Spelke, & Streri, 2009; Xu & Spelke, 2000), and by the first few months of life, they can perform simple numerical additions, subtractions, and comparisons (Brannon & Van de Walle, 2001; McCrink & Wynn, 2004, 2007; Wynn, 1992). To account for these competences, current theories grant infants two core systems capable of encoding numerical information (Carey, 2009; Feigenson et al., 2004; Hyde, 2011). These two systems are associated with infants' numerical capacities with large and small sets, respectively. First, infants can represent, compare, and perform arithmetic operations on large approximate numerosities. Second, infants can track small sets of up to 3 or 4 objects, and through these attentional abilities, they can solve simple arithmetic tasks involving small exact numbers of objects.

Yet, infants' sensitivity to number shows striking limitations when compared to the power of the simplest mathematical numbers: the integers, or "natural numbers." In the large number range (beyond 3 items), infants' discrimination of numerosities is approximate and follows Weber's law: numerosities can be discriminated only if they differ by a minimal ratio (Xu, Spelke, & Goddard, 2005). The same imprecise representations are found in young children and even in educated adults, when they are prevented from counting (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza et al., 2010). Because of this limitation, numerosity perception fails to capture two essential properties that are central to formalizations of the integers: the relation of exact numerical equality and the successor function (Izard, Pica, Spelke, & Dehaene, 2008; Leslie, Gelman, & Gallistel, 2008). The relation of exact numerical equality grounds integers in set-theoretic constructions: two sets are equinumerous if and only if their elements can be placed in perfect one-to-one correspondence (this is Hume's principle). The successor function, on the other hand, is the initial intuition underpinning the Peano–Dedekind axioms: here the integers are generated by successive additions of one, i.e., by the iteration of a successor operation.

Theories diverge with regards to the origins of the concept of exact number in children's development. Some have proposed that exact number is innate, either because the properties of exact number are built into the system of analog mental magnitude (Gelman & Gallistel, 1986), or because there is a separate system giving children an understanding of exact equality and/or of the successor function (Butterworth, 2010; Hauser, Chomsky, & Fitch, 2002; Leslie et al., 2008; Rips, Bloomfield, & Asmuth, 2008). For example, Leslie et al. proposed that children have an innate representation of the exact quantity ONE that can be used iteratively to generate representations of exact numbers. In the same vein, Frank, Everett, Fedorenko, and Gibson (2008) and Frank, Fedorenko, Lai, Saxe, and Gibson (2011) proposed that humans can represent one-to-one correspondence non-symbolically and know intuitively that perfect one-to-one correspondence entails exact numerical equality. Other theorists have proposed that the concept of exact number is constructed, and that symbols such as tally marks, numerical expressions in natural language, abacus configurations, or other symbolic devices play a crucial role in this process (Bialystok & Codd, 1997; Carey, 2009; Cooper, 1984; Fuson, 1988; Klahr & Wallace, 1976; Mix, Huttenlocher, & Levine, 2002; Piantadosi, Tenenbaum, & Goodman, 2012; Schaeffer, Eggleston, & Scott, 1964; Spelke, 2003). For example, Carey (2009) proposed that children construct the natural numbers by (1) learning the ordered list of count words as a set of uninterpreted symbols, then (2) learning the exact meanings of the first three or four count words, mapping the words to representations of 1–4 objects that are attended in parallel, and finally (3) constructing an